Model reference composite learning control without persistency of excitation

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Abstract: Parameter convergence is desirable in adaptive control as it brings several attractive features, including accurate online modelling, exponential tracking, and robust adaptation without parameter drift. However, a strong persistent-excitation (PE) condition must be satisfied to guarantee parameter convergence in the conventional adaptive control. This study proposes a model reference composite learning control strategy to guarantee parameter convergence without the PE condition. In the composite learning, an integral at a moving-time window is applied to construct a prediction error, an integral transformation is derived for avoiding the time derivation of plant states in the calculation of the prediction error, and both the tracking error and the prediction error are applied to update parametric estimates. Global exponential stability of the closed-loop system is established under an interval-excitation condition which is much weaker than the PE condition. Compared with a concurrent learning technique that has the same aim as this study, the proposed composite learning technique avoids the usage of singular value maximisation and fixed-point smoothing resulting in a considerable reduction of computational cost. Numerical results have verified effectiveness and superiority of the proposed control strategy.

1 Introduction

For non-linear systems with parametric uncertainties, adaptive control had been well established at the end of the last century [1–3]. Yet, the advancement of adaptive control still kept great attraction, where some survey papers in the past decade can be referred to [4–12]. In particular, model reference adaptive control (MRAC) is a popular adaptive control architecture which aims to make an uncertain dynamical system behave like a chosen reference model. The way of parameter estimation in adaptive control gives rise to two different schemes, namely indirect and direct schemes [3]. In the indirect scheme, plant parameters are estimated online for the calculation of controller parameters, whereas in the direct scheme, the plant model is parameterised in terms of controller parameters that are estimated directly without plant parameter estimation. Generally in the adaptive control, only asymptotic convergence of tracking errors can be achieved, and parameter convergence cannot be guaranteed without a persistent-excitation (PE) condition [1–3]. Nevertheless, the PE condition is very strong and often infeasible in practice [13–15].

Composite adaptive control (CAC) is an integrated direct and indirect adaptive control strategy which aims to achieve better trajectory tracking and parameter estimation through faster and smoother parameter adaptation [16]. In the CAC, prediction errors are generated by identification models, and both tracking errors and prediction errors are applied to update parametric estimates. The superior control performance of CAC has been demonstrated in many studies, where some typical results during the last decade can be referred to [17–27]. In [17], a composite MRAC approach was applied to control longitudinal movement of an aircraft. In [18], a multiple-models switching technique was incorporated into composite MRAC to improve the transient performance of adaptive control. The CAC was extended to a general class of uncertain non-linear systems in [19]. In [20], a generic composite MRAC architecture was developed for a class of multiple-input multiple-output (MIMO) uncertain non-linear systems. In [21], a posicast CAC framework is proposed for a class of linear systems with known input delay. The CAC approaches of [20, 21] were also applied to control longitudinal movement of aircrafts. In [22], a CAC-based synchronisation scheme was proposed for bilateral tele-operation systems. Note that only structured linear-in-the-parameters (LIP) uncertainties are considered in all above-mentioned CAC approaches. In [23], a novel CAC scheme with a robust error-sign integral technique was developed for a general MIMO Euler–Lagrange system with mixed structured and unstructured uncertainties. The extensions of CAC to non-linear systems with functional uncertainties can be referred to [24–27].

An emerging Q-modification technique was proposed to construct an alternative CAC scheme in [28]. Differing from the conventional CAC that utilises identification models and linear filters to generate prediction errors, the Q-modification-based CAC integrates the system dynamics in a moving-time window to generate prediction errors. The time-interval integral is useful for utilising data recorded online to improve parameter estimation. Note that only matched uncertainties are considered in all above-mentioned CAC approaches. Based on the Q-modification technique, an integrator backstepping CAC approach and a dynamic surface CAC approach were developed for a class of strict-feedback non-linear systems with mismatched parametric uncertainties in [29] and [30], respectively. Although better tracking and parameter estimation can be obtained in all above-mentioned CAC approaches, the PE condition still has to be satisfied to guarantee parameter convergence.

Learning is a fundamental feature of autonomous intelligent behaviour [31], and it is reflected by parameter convergence in adaptive control [32]. The benefits brought by parameter convergence include accurate online modelling, exponential tracking, and robust adaptation without parameter drift [15]. An emerging concurrent learning technique provides a promising way for achieving parameter convergence in MRAC without the PE condition [13–15]. The difference between the concurrent learning and the composite adaptation lies in the construction of prediction errors. In the concurrent learning, a dynamic data stack constituted by data recorded online is used in constructing prediction errors, and exponential convergence of both tracking errors and estimation errors is obtained if regression functions are excited over a time interval such that sufficiently rich data are recorded in the data stack. However, in this innovative design, an exhaustive search algorithm must be applied to the data stack to maximise its singular
value, and a fixed-point smoothing technique must be applied to estimate time derivatives of plant states for the calculation of prediction errors. These deficiencies inevitably increase computational cost of the entire control algorithm.

This paper focuses on model reference composite learning control (MRCLC) for a class of parametric uncertain affine nonlinear systems, where a novel composite learning technique is developed to guarantee parameter convergence without the PE condition. The design procedure of the MRCLC is as follows:

First, the classical MRAC law is presented to facilitate control synthesis; second, a modified modelling error that utilises data recorded online is defined as the prediction error; third, an integral transformation is derived to avoid the time derivation of plant states in the calculation of the prediction error; fourth, both the tracking error and the prediction error are applied to update parametric estimates; finally, global exponential stability of the closed-loop system is established by an interval-excitation (IE) condition which is much weaker than the PE condition. The significance of this study is that the deficiencies of the concurrent learning MRAC are completely avoided by the proposed MRCLC resulting in a considerable reduction of computational cost. This study is based on our previous works [33–36], where the simulation results with deep discussions are provided in this study.

The notations of this paper are relatively standard, where \( \mathbb{R} \), \( \mathbb{N} \), \( \mathbb{R}^+ \), \( \mathbb{R}^n \), and \( \mathbb{R}^{n \times m} \) denote the spaces of numbers, real numbers, positive real numbers, real n-vectors and real \( n \times m \)-matrices, respectively. \( L_\infty \) denotes the space of bounded signals, \( \| x \| \) denotes the Euclidean-norm of \( x \), \( \Omega := \{ x : \| x \| \leq c \} \) denotes the ball of radius \( c \), \( \min \{ \cdot \} \) denote the minimum and maximum functions, respectively, \( \lambda_{\min}(A) \) and \( \lambda_{\max}(A) \) denote the minimum and maximum eigenvalues of \( A \), respectively, \( (\cdot, \cdot) \) represents the inner product, \( \parallel \cdot \parallel \) denotes the Euclidean-norm of \( \cdot \), \( \int_{t_0}^{t_1} \) denotes an integral, \( \mathcal{L}_{\infty} \) denotes the space of functions for which all \( k \)-order derivatives exist and \( \mathcal{C}^1 \) represents the space of functions for which all \( k \)-order derivatives exist and are continuous, where \( c \in \mathbb{R}^+ \), \( \epsilon \in \mathbb{R} \), \( x \in \mathbb{R}^n \), \( A \in \mathbb{R}^{n \times n} \), \( i = 1 \) to \( n \), and \( n, m, k \in \mathbb{N} \). Note that in the subsequent sections, the arguments of a function may be omitted while the context is sufficiently explicit.

## 2 Problem formulation

For simplifying presentation, consider a class of SISO affine nonlinear systems with LIP uncertainties as follows [13]:

\[
x(t) = Ax(t) + b(f(x(t)) + u(t)) \tag{1}
\]

with \( A \in \mathbb{R}^{n \times n} \) and \( b := [0, \ldots, 0, 1]^T \), where \( x(t) := [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n \) is a vector of plant states, \( u(t) \in \mathbb{R}^n \) is a vector of control inputs, and \( f(x) : \mathbb{R}^n \rightarrow \mathbb{R} \) is a \( \mathcal{C}^1 \) model uncertainty. The discussion about the extension to wider classes of nonlinear systems can be referred to [37, Remark 1]. A reference model that characterises the desired response is given by

\[
x^* = Ax^* + b_x^* \tag{2}
\]

with \( b_x := [0, \ldots, 0, b_x^*] \in \mathbb{R}^n \), where \( A_x \in \mathbb{R}^{n \times n} \) is a strictly Hurwitz matrix, \( x_x(t) := [x_{x_1}(t), x_{x_2}(t), \ldots, x_{x_n}(t)]^T \in \mathbb{R}^n \) is a vector of reference states, and \( r(t) \in \mathbb{R}^n \) is a bounded reference signal. This study is based on the facts that \( x \) is measurable, \( (A, b) \) is controllable, and \( f(x) \) is linearly parameterisable as follows [13]:

\[
f(x) = W^T \Phi(x) \tag{3}
\]

where \( W^T \in \Omega_{r_x} \subset \mathbb{R}^n \) is a vector of unknown constant parameters, \( \Phi(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n \) is a vector of known regression functions, and \( c_w \in \mathbb{R}^+ \) is a known constant. The following definitions are introduced for facilitating control synthesis [13].

**Definition 1:** A bounded signal \( \Phi(t) \in \mathbb{R}^N \) is of IE over \( [T_e - t_p, T_e] \) if there are constants \( T_p, T_e \in \mathbb{R}^+ \) so that \( \int_{T_e - T_p}^{T_e} \Phi(t) \Phi^T(t) \, dt \geq c_1 \).

**Definition 2:** A bounded signal \( \Phi(t) \in \mathbb{R}^N \) if of PE iff there are constants \( T_p, T_e \in \mathbb{R}^+ \) so that \( \int_{T_e - T_p}^{T_e} \Phi(t) \Phi^T(t) \, dt \geq c_1 \).

Let \( x_{nu}(t) := [x_x(t), r(t)]^T \) be an augmented reference signal, and \( \dot{W}(t) \in \mathbb{R}^N \) be an estimate of \( \dot{W} \). Define the tracking error \( e(t) := x(t) - x_x(t) \), and the parameter estimation error \( \dot{W}(t) := \dot{W} - \dot{W}(t) \). The objective of this study is to design a proper control law \( u \) such that exponential convergence of both \( e \) and \( \dot{W} \) can be guaranteed by the IE condition in Definition 1.

## 3 Composite learning control design

### 3.1 Review of previous results

A MRAC law is presented as follows [3]:

\[
u = -k_e e + k_x x_x - W^T \dot{\Phi}(x) \tag{4}
\]

where \( u_{na} \) is a proportional–derivative (PD) feedback controller, \( u_{na} = \) a feedforward controller, \( u_{na} \) is an adaptive controller, \( k_e \in \mathbb{R}^n \) and \( k_r \in \mathbb{R}^{n+1} \) are control gains, and the choice of \( k_r \) satisfies

\[
k_r \dot{x}_r = (A_r - \Lambda)x_r + b_r r. \tag{5}
\]

Substituting (4) and (5) into (1), one obtains the tracking error dynamics as follows:

\[
e(t) = \dot{A}P + PA = -Q. \tag{6}
\]

Let an adaptive law of \( \dot{W} \) be as follows:

\[
\dot{W} = \gamma P(e^T P \Phi(x)) \tag{8}
\]

where \( \gamma \in \mathbb{R}^+ \) is a learning rate, and \( \mathcal{P}(\cdot) \) is a projection operator in the following form [3]:

\[
\mathcal{P}(\cdot) = \begin{cases} \cdot & \text{if } \parallel W \parallel < c_w \text{ or } \parallel W \parallel = c_w & \parallel W \parallel \cdot \leq 0 \\ -\bar{W}W^T/\parallel W \parallel_2, & \text{otherwise} \end{cases}
\]

Choose a Lyapunov function candidate

\[
V(z) = e^T Pe/2 + \dot{W}^T W/(2\gamma) \tag{9}
\]

with \( z := [e^T, \dot{W}^T] \in \mathbb{R}^{n+N} \) for the closed-loop dynamics composed of (6) and (8). It follows from the standard MRAC result in [3] that if \( \dot{W}(0) \in \Omega_{r_x} \) and \( \Phi \) meets the PE condition in Definition 2, then the closed-loop system achieves global exponential stability in the sense that both \( e(t) \) and \( \dot{W}(t) \) exponentially converge to 0.
To relax the PE condition for parameter convergence in MARC, a concurrent learning law of ̇W is proposed as follows [13]:

\[ \dot{W} = P(e^T P b \Phi(x) + \sum_{j=1}^{n} \epsilon_j \Phi^T(x_j)) \]  

(10)

where \( j \) denotes a certain epoch, \( p \geq N \) denotes a number of stored data, and \( \epsilon_j := \tilde{W}^T \Phi(x_j) \) denotes a modelling error which is regarded as the prediction error calculated by the exhaustive search algorithm must be applied to the data stack \( Z \). The following lemma from [13] shows the stability result of the concurrent learning MARC.

**Lemma 1:** Consider the system (1) driven by the control law (4) with (10), where the control gain \( k_c \) is selected to satisfy (5), and the control gain \( k_c \) is selected to make \( A \) in (6) strictly Hurwitz. If it has \( \tilde{W}(0) \in \Omega_{\epsilon,0} \) and rank \( (Z) = N \), then the closed-loop system achieves global exponential stability in the sense that both \( e(t) \) and \( \tilde{W}(t) \) exponentially converge to 0.

**Remark 1:** The concurrent learning achieves parameter convergence in MRAC via the condition rank \((Z) = N\) which is equivalent to the IE condition in Definition 1, where its prominent feature is that data are available at any time across the control process can be incorporated into the adaptive learning (10). However, this innovative technique has some deficiencies as follows: (i) a non-exhaustive search algorithm must be applied to the data stack \( Z \) to maximise its singular value; and (ii) a fixed-point smoothing technique must be applied to estimate \( \Phi(x) \) such that the prediction errors \( \epsilon_j \) in (11) are calculable. These deficiencies significantly increase computational cost of the entire control algorithm.

### 3.2 Composite learning control scheme

This section aims to eliminate the drawbacks of the concurrent learning. For facilitating presentation, define

\[ \Theta(t) := \int_{t_{\tau_d}}^{t} \Phi(x(r)) \Phi^T(x(r)) dr \]  

(12)

in which \( \tau_d \in \mathbb{R}^+ \) is an integral duration. Then, the IE condition in Definition 1 can be rewritten as \( \Theta(T_d) \geq \gamma \) with \( \tau_d \in \mathbb{R}^+ \), where \( \sigma \) is regarded as an exciting strength. For a certain control problem with a given \( \tau_d \in \mathbb{R}^+ \), the epoch \( T_d \) that satisfies the IE condition is usually not unique, and the corresponding \( \sigma \) can be time-varying.

To estimate the maximal exciting strength, \( \sigma(t) := \max_{e \in \mathbb{R}^+} \{ \sigma(r) \} \) is proposed as follows:

\[ \hat{\sigma}(t) := \max_{e \in \mathbb{R}^+} \{ \sigma(r) \} \]  

(13)

as the prediction error. Then, it is convenient to give a composite learning law of \( \tilde{W} \) as following:

\[ \hat{\tilde{W}} = P(e^T P b \Phi(x) + \gamma k_c e) \]  

(14)

in which \( k_c \in \mathbb{R}^+ \) is a weight factor. A block diagram of the complete MRCLC scheme is presented in Fig. 2. To calculate \( \theta W \) in (13), (3) is substituted into (1) and both sides of (1) are multiplied by \( \Phi(x) b^T \) so that

\[ \Phi(x) = \Phi(x)(b^T \Lambda x + \Phi(x)^T W + u) \]  

(15)

Integrating both sides of (15) over \([t - \tau_d, t]\) and applying (12) to the resulting expression, one obtains

\[ \theta W = \int_{t - \tau_d}^{t} \Phi(x)(x_n - b^T \Lambda x - u) \ dr \]  

(16)

where the first term of the integral part is

\[ \int_{t - \tau_d}^{t} \Phi(x)x_n \ dr = \int_{t}^{t} \Phi(x)x_n \ dr \]  

(17)

Let \( \tilde{x}_i := [x_{i,1}, x_{i,2}, \ldots, x_{i,n}]^T \) and \( \tilde{x}_n := [x_{n,1}, x_{n,2}, \ldots, x_{n,n-1}]^T \). The following lemma shows how to calculate the two terms at the right side of (17) without the usage of the immeasurable \( x_n \).

**Lemma 2:** \( \int_{t}^{t} \Phi(x) \tilde{x}_n \ dr \) in (17) can be calculated as follows:

\[ \int_{0}^{t} \Phi(x) \tilde{x}_n \ dr = \int_{0}^{t} \Phi(\tilde{x}_n) \ d\tilde{x}_n - \int_{0}^{t} \Psi(x) \ dr \]  

(18)

with \( \Psi(x) \) being defined by

\[ \Psi(x) := \int_{0}^{t} \frac{\partial \Phi(\tilde{x}_n)}{\partial \tilde{x}_n} \ d\tilde{x}_n. \]

where \( \partial \Phi(\tilde{x}_n)/\partial \tilde{x}_n \) results in a \( N \times (n - 1) \)-dimension Jacobian matrix from the vector calculus.
Theorem 1: The results of the closed-loop system comprised of (6) and (14) in the sense that all closed-loop signals are uniformly bounded, above expression, noting one also gets (18). □

Remark 2: Although the proposed composite learning also utilises data recorded online, it is fundamentally different from the concurrent learning due to the following two aspects: (i) the time-interval integral in (12) is applied to construct the prediction error \( \varepsilon \) in (13) such that the singular value maximisation in the concurrent learning is not needed; and (ii) the integral transformation in (18) is derived to calculate the prediction error \( \varepsilon \) in (13) without the usage of the immeasurable \( x_s \) such that the fixed-point smoothing in the concurrent learning is avoided. Hence, the proposed composite learning eliminates the two major deficiencies of the concurrent learning described in Remark 1 so that it deserves to perform better than the concurrent learning. The superior performance of the composite learning compared with the concurrent learning will also be demonstrated by illustrative examples in Section 4.

3.3 Stability and convergence analysis

The theorem establishes the stability and convergence results of the closed-loop system comprised of (6) and (14).

Theorem 1: Consider the system (1) driven by the control law (4) with (14), where the control gain \( k_c \) is selected to satisfy (5), and the control gain \( k_s \) is selected to make \( A \) in (6) strictly Hurwitz. If \( W(0) \in \Omega_{w} \) and \( \Theta(T) \geq \sigma I \) for some constants \( T_{e}^{r}T_{p} \sigma \in \mathbb{R}^{+} \), then the closed-loop system achieves global exponential stability in the sense that all closed-loop signals are uniformly bounded, \( \forall \ t \geq 0 \), and both \( e(t) \) and \( W(t) \) exponentially converge to \( 0 \), \( \forall \ t \geq T_{e} \).

Proof: First, consider the control problem at \( t \in [0, \infty) \). Choose the Lyapunov function candidate \( V \) in (9) for the closed-loop system. The time derivative of \( V \) along (6) is as follows:

\[
V = -e^{T}Qe/2 + \bar{W}^{T}(e^{T}Pb\Phi(x) - \bar{W}/\gamma)
\]

where (7) is utilised to obtain the above result. Applying (14) to the above expression, noting \( \bar{W}(0) \in \Omega_{w} \) and using the projection operator result in [3], one gets \( \dot{W}(t) \in \Omega_{w} \), \( \forall t \geq 0 \) and

\[
\dot{V} \leq -e^{T}Qe/2 - k_{m}W^{T}e, \quad \forall t \geq 0. \quad (20)
\]

Noting the definition of \( \varepsilon \) in (13), one gets \( W^{T}e \geq 0 \) so that

\[
\dot{V} \leq -e^{T}Qe/2, \quad \forall t \geq 0.
\]

Thus, one immediately gets \( V \leq 0, \forall t \geq 0 \), which implies the closed-loop system is stable in the sense of \( e, W \in L_{\infty} \). As \( V \leq 0 \) is satisfied, \( \forall x(0) \in \mathbb{R}^{n} \), and \( V \) in (9) is radially unbounded (i.e. \( V(\bar{x}) \rightarrow \infty \) as \( \| \bar{x} \| \rightarrow \infty \)), the stability is global. Using \( e, W \in L_{\infty} \), one also gets \( x, W, \Phi, \varepsilon, u \in L_{\infty} \) from their definitions. Thus, all closed-loop signals are uniformly bounded, \( V \geq 0 \).

Second, consider the control problem at \( t \in [T_{e}, \infty) \). As there exist \( T_{e}^{r}T_{p} \sigma \in \mathbb{R}^{+} \) such that \( \Phi(T) \geq \sigma I \), i.e. the bounded \( \Phi(t) \) is of IE over \( [T_{e}, \infty) \), it is obtained from (20) that

\[
V \leq -e^{T}Qe/2 - k_{e}W^{T}e, \quad \forall t \geq T_{e} \quad (21)
\]

with \( k_{e} = \max_{t \in [T_{e}, \infty)} \sigma(t) \). It follows from (9) and (21) that

\[
\dot{V}(t) \leq -k_{e}V(t), \quad \forall t \geq T_{e}.
\]

4 Illustrative examples

4.1 Example 1: Inverted pendulum

Consider the following inverted pendulum model [13]:

\[
x = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} (W^{T} \Phi(x) + u)
\]

where \( W = [1, -1, 0.5, 0]^{T} \) and \( \Phi(x) = [\sin x_{1}, x_{1}x_{2}, e^{x_{2}}x_{1}]^{T} \), where \( x_{1} \) (rad) is the angular position of the pendulum, \( x_{2} \) (rad/s) is the angular velocity of the pendulum, and \( u \) (V) is the control input voltage. The reference model is given by

\[
x_{r} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x_{r} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

where \( x(0) = x_{r}(0) = [1, 1]^{T}, r = 1 \) at \( t = 20, 25 \), and \( r = 0 \) at \( t \in [0, 20) \cup (25, \infty) \).

The parameters selection of the proposed control law (4) follows that of [13], where the details are given as follows: First, solve (5) to get \( k_{c} = [-1, -2, 1]^{T} \); second, select \( k_{s} = [1, 1, 1]^{T} \) such that \( A \) is strictly Hurwitz; third, solve (7) with \( Q = \text{diag}(10, 10) \) to obtain \( P \); fourth, set \( T_{e} = 5 \) s in (13), and finally, set \( \gamma = 3.5, k_{m} = 6 \) and \( c_{m} = 5 \) in (14).

Simulations are carried out in MATLAB software running on Windows 7 and an Intel Core i7-4510U CPU, where the Solver is chosen as fixed-step ode 1 with a step size being 1 ms and the other settings being defaults. The classical MRAC in [3], the model reference CAC (MRCAC) with \( Q \)-modification in [28], and the concurrent learning MRAC (CLMRAC) in [13] are selected as baseline controllers, where the other settings of the CLMRAC are kept the same as those in [13], and shared parameters of all controllers applied are set to be same values for fair comparison.

Simulation trajectories by the classical MRAC, the MRCAC, the CLMRAC and the proposed MRLC are depicted in Figs. 3–6, respectively. For the control performance, it is shown that the plant state \( x \) follows its desired signal \( x_{r} \) closely with a smooth control input \( u \) for each controller applied, the MRCAC achieves the worst tracking accuracy (see Fig. 4a), the CLMRAC exhibits a large tracking error \( e \) at the initial control stage (see Fig. 5a), and the proposed MRLC achieves the best tracking accuracy (see Fig. 6a). For the learning performance, it is observed that IE instead of PE occurs in this case, the MRAC does not show any parameter convergence (see Fig. 3b), the MRLC shows better parameter estimation than the MRAC, but still does not achieve parameter convergence (see Fig. 4b), and both the CLMRAC and the proposed MRLC achieve fast parameter convergence even the IE is short and weak (see Figs. 5b and 6b). Note that only the current maximal exciting strength \( \sigma_{e} \) is shown for the CLMRAC (see Fig. 5b) due to its different definition of prediction errors.

A performance comparison among all controllers applied is depicted in Fig. 7, which verifies the best tracking and learning performance. \( i \leq j \).

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performances of the proposed MRCLC. More specifically, due to the partial asymptotic property with respect to $\gamma$, the MRAC shows the fastest tracking at the initial control stage, yet spends more than 12 s to readapt a new control task (i.e. the step command at $t = 20$ s) resulting in a sharp degradation of tracking accuracy within $t \in [20, 32]$ s; the MRCAC performs even worse than the MRAC as it tries but fails to minimise the estimation error (see also Fig. 4); due to the exponential convergence of both $\gamma$ and $\tilde{\gamma}$, both the CLMRAC and the proposed MRCLC sacrifice some tracking accuracy at the initial control stage for the convergence of $\tilde{\gamma}$ during $t \in [0, 5.8]$ s, keep high tracking accuracy after the convergence of $\tilde{\gamma}$ during $t \in [5.8, 20]$ s, and demonstrate high insensitivity to the new control task during $t \in [20, \infty)$ s. The stages of learning, storage and reusage of the plant knowledge (i.e. the uncertainty $\gamma^*$) are also simply indicated in Fig. 7. Compared with the CLMRAC, the superiority of the proposed MRCLC lies in the much higher initial tracking accuracy and the much faster executing speed. It is shown in Fig. 8 that the proposed MRCLC performs over 12 times’ faster than the CLMRAC in this example.

4.2 Example 2: aircraft wing rock

Consider the following aircraft wing rock model [15]:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (W^\gamma \Phi(x) + bu)$$

with $\Phi(x) = [1, x_1, x_2, x_3, x_4, x_5, x_6, x_7]^T$, where $x_1$ (rad) is the aircraft roll angle, $x_2$ (rad/s) is the roll rate, $u$ (rad) is the aileron control input, $b \in \mathbb{R}$ is a known control gain, and $W^\gamma$ is a vector of unknown coefficients related to angle of attack. For simulation, let $x(0) = [68\pi/180, -57\pi/180]^T$, $b = 3$ and $W^\gamma = [0.8, 0.2314, 0.6918, -0.6245, 0.0095, 0.0214]^T$ and

$$x_r = \begin{bmatrix} 0 \\ 1 \\ -1 \\ -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \gamma$$

where $x_r(0) = x(0)$, $r = 57\pi/180$ at $t \in [15, 17]$ s, $r = -57\pi/180$ at $t \in [25, 27]$ s and $r = 0$ for the other time. The parameters selection of the proposed control law (4) with (14) in this example is the same as that of Example 1 except $k_r = [-1, -1, 1]^T$, $k_\gamma = [1, 1]^T$, $\tau_D = 10$ s, and $\gamma = k_w = 5$. Simulation trajectories by the classical MRAC and the proposed MRCLC are depicted in Figs. 9 and 10, respectively, and simulation trajectories by the MRCAC and the CLMARC are not presented here to save page space due to their dissatisfactory performances shown in Example 1. For the control performance, it is observed that the MRAC achieves satisfactory tracking of $x_1$ and exhibits oscillations at $x_2$ resulting in oscillations at $u$ (see Fig. 9a), whereas the MRAC achieves better tracking without oscillations at $x_1$, $x_2$ and $u$ (see Fig. 10a). For the learning performance, it is observed that the MRAC does not show any parameter convergence (see Fig. 9b), and the MRCLC achieves fast parameter convergence even the IE is weak (see Fig. 10b).

Simulation trajectories by the conventional MRAC and the proposed MRCLC under 40 dB measurement noise are given in Figs. 11 and 12, respectively, to verify robustness against measurement noise of the applied controllers, where qualitative analysis of these results is the same as that of the noise-free case.
except that chattering at the control input $u$ occurs for both the controllers. In addition, performance comparisons of the two controllers under both the noise-free and noisy-measurement cases are given in Fig. 13 to further demonstrate the performance improvement of the proposed MRCLC. Furthermore, a comparison of simulation speeds between the two controllers is given in Fig. 8 to verify high computational efficiency of the proposed MRCLC, where it is shown that the proposed MRCLC performs over 17 times faster than the CLMRAC in this example.

Fig. 5  Simulation trajectories by the CLMRAC of [13] in Example 1
(a) Control performance, (b) Learning performance

Fig. 6  Simulation trajectories by the proposed MRCLC in Example 1
(a) Control performance, (b) Learning performance

Fig. 7  Performance comparison of all controllers in Example 1

Fig. 8  Comparison of simulation speeds between two learning techniques

5 Conclusion
In this paper, a MRCLC strategy has been successfully developed to guarantee fast parameter convergence at the absence of the PE condition. The significance of the proposed approach is that it completely eliminates the major deficiencies of the concurrent learning resulting in a sharp decrease of computational cost. Two illustrative examples have demonstrated the best control and learning performances of the proposed approach compared with existing approaches. Specifically, it is observed that the proposed MRCLC executes much faster with much higher initial tracking accuracy than the concurrent learning MRAC. Further work on the composite learning, including the consideration of internal/external perturbations and the extension to wider classes of uncertain non-linear systems, is currently under investigation.
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References


Fig. 12 Simulation trajectories by the proposed MRCLC in Example 2 with measurement noise
(a) Control performance, (b) Learning performance

Fig. 13 Performance comparisons of two controllers in Example 2
(a) Without measurement noise, (b) With measurement noise


