Continuous Tracking Control for a Compliant Actuator With Two-Stage Stiffness
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Abstract—Emerging applications of robots with direct physical interactions with humans have led to the development of a variety of series elastic actuators (SEAs) which are compliant, force controllable, and back drivable. The performance of current SEAs is mainly dependent on the specific stiffness of the spring. In our previous work, a compliant actuator with two-stage stiffness has been designed to overcome the performance limitations in current SEAs. The key novelty is that a low-stiffness spring and a high-stiffness spring are employed instead of a single spring in current SEAs, which has the advantages of high fidelity, low output impedance, and also large force range and bandwidth. In this paper, a tracking control scheme is proposed for the compliant actuator with two-stage stiffness. Although the overall stiffness is discontinuous, the proposed controller is designed by integrating different control modes for two springs into a single one. The transition between control modes is smooth and embedded inside the controller, and it is also automatically realized by monitoring the output force of the actuator. The stability and convergence of the closed-loop system are analyzed, and experimental results are presented to demonstrate the effectiveness of the proposed control scheme.

Note to Practitioners—An SEA is developed by placing an elastic element into the actuator; this elasticity gives SEAs several unique properties including low mechanical output impedance, tolerance to impact loads, and passive mechanical energy storage, which makes it suitable for human–robot interaction. The performance of existing SEAs is highly dependent on the stiffness of a single spring. To overcome the limitations, a novel SEA with two-stage stiffness was proposed in our previous work. This paper suggests a continuous tracking control method for the proposed compliant actuator. Although the overall stiffness is discontinuous, the transition between different control modes for two springs is smooth and automatically realized. Experimental results show that the output force of the actuator is bounded. In future research, uncertainties in actuator dynamics will be considered, such that system identification or calibration is not required.

Index Terms—Series elastic actuator (SEA), smooth and automatic transition, tracking control, two-stage stiffness.

I. INTRODUCTION

The demand for service robots [1]–[3] and assistive and rehabilitation robots [4]–[6] in both domestic and hospital settings has been increasing significantly, due to the rapidly aging populations in most developed nations. In those applications where the robots are required to directly interact with humans, safety is always the most critical concern. The safe and human-friendly robotic applications have become key drivers for the research on compliant actuators. In general, the compliant actuators used in human–robot interactions are able to achieve force control, impedance control, and back-drivability [7], [8]. A review of the comparison of different compliant actuators can be found in [9]. Among the various designs of compliant actuators for human-friendly robotic applications [10]–[12], the most commonly adopted actuator is the series elastic actuator (SEA).

The first SEA design was proposed by Pratt and Williamson [13], where an elastic element was placed in series with the motor and gear transmission. The basic components of SEAs proposed in [13] consist of a motor, a gear transmission, a set of linear springs, and a sensor measuring the deflection of the spring, and the force output of SEA is calculated using Hooke’s law. Later, Robinson [14] carried out further studies on the modeling and control analysis of the SEA in his Ph.D. research. By introducing an elastic element between the load and the geared motor, the inertia and nonlinear frictions of the motor and the transmission are decoupled from the load, and external impacts and shocks are isolated from the gear transmission. It also makes the force sensing and control problem into a position sensing and control problem. Therefore, in human-friendly robotic applications, SEAs are known to offer a range of advantages over stiff actuators, such as high force/torque controllability and fidelity, low output impedance, back-drivability, and tolerance to shock and impacts.

Inspired by the original work [13], [14], much progress has been achieved in the development and applications of SEAs [15]–[18]. The SEA was implemented in the RobotKnee system [15] to achieve low impedance and high force fidelity. In [16], a Bowden-cable-based SEA was developed for the LOPES robotic exoskeleton system. A rotary SEA was designed in [17] to provide partial support to knee flexion/extension during overground walking. A compact and lightweight prismatic SEA was reported in [18], which is based on a piston-style ball screw support mechanism and a concentric compliant element. However, existing SEAs commonly face a fundamental limitation, i.e., the fixed stiffness
of the elastic element as discussed by Pratt et al. [15] and Robinson et al. [19]. The performance of SEAs largely depends on the selection of the stiffness of a single spring [19]. A soft spring produces high fidelity, low output impedance, and reduces stiction, but also limits the force range and the bandwidth. On the other hand, a stiff spring increases the bandwidth, but reduces force fidelity, leading to low intrinsic compliance and backdrivability, and bulky and heavy systems.

To overcome the aforementioned limitations of conventional SEAs, a new compact compliant actuator was first proposed in [20]. The key novelty is that the combination of a low-stiffness spring and a high-stiffness spring is employed instead of a single spring, where the two springs are utilized in the low-force range and the high-force range of the compliant actuator, respectively. Based on the two-stage stiffness, the compliant actuator in [20] is able to generate high force fidelity and low output impedance, and also extend the force range and the bandwidth. Therefore, the performance of SEA is significantly improved.

However, the overall stiffness of the compliant actuator is discontinuous, due to the hard switching of two springs. While several control methods have been proposed for SEAs [20]–[25], the issue of discontinuity cannot be directly addressed using the existing control techniques. A force control method was reported for the compliant actuator with two-stage stiffness in [20], where two controllers were developed and switched in low-force range or high-force range, respectively. This paper considers the position control problem of the compliant actuator and presents a continuous trajectory-tracking control scheme. Instead of designing multiple controllers and switching between them, the proposed method smoothly and stably integrates two control modes into a single controller. Although the overall stiffness is discontinuous, the proposed controller is continuous without any hard switching, and the transition between different control modes is automatically realized by monitoring the output force of the actuator. The stability and convergence of the overall closed-loop system are analyzed, with consideration of the transition between control modes. Experimental results are presented to demonstrate the effectiveness of the proposed control method.

II. COMPLIANT ACTUATOR WITH TWO-STAGE STIFFNESS

A. Working Principle

In our previous work, a compliant actuator with two-stage stiffness has been designed, as shown in Fig. 1(a). It mainly consists of a servomotor, two rotary encoders, a torsional spring, a set of linear springs, a ball screw, and a potentiometer. The motion from the motor is first transmitted to the ball screw via the torsional spring and a spur gear, which converts the rotatory motion of the shaft into the linear motion of the ball screw nut. Then, the motion of the nut is transmitted to the output carriage through the linear springs. One encoder is installed on the motor to measure the angular displacement of the motor and the torsional spring, while the other encoder is employed to measure the angular displacement of the ball screw. The linear potentiometer is installed to measure the displacement of the linear spring.

In this design, the stiffness of the linear spring is chosen to provide the average output force, and the linear spring is soft, small, and lightweight. A very small torsional spring is then used at the output end of the motor and its effective spring stiffness is more than 100 times that of the linear spring. Due to the difference in spring stiffness, when the linear spring plays a dominant role, the torsional spring behaves like a rigid link. When the torsional spring is activated, the soft linear spring is almost saturated and the force control is based on the torsional spring. Therefore, we can achieve a much smaller physical size compared with existing SEA designs.

Fig. 1(b) shows the prototype of the actuator. The actuator is designed to be able to provide up to 55 Nm output torque. A Maxon DC brushless motor (EC 4-pole 120 Watt 36 V) is employed due to its light weight (0.175 kg) and low moment of inertia. The ball screw selected from Eichenberger Gewinde AG has a pitch of 2 mm/rev and can output over 1100 N force. The stiffness of the linear springs is 24 N/mm, while the working stroke is 10 mm. The linear springs provide a maximum output force of 240 N. The stiffness of the torsional spring is 0.29 Nm/rad and the deflection range is 72 degrees. The resolution of the incremental rotary encoder is 1024 lines/rev, and the total mass of the actuator is less than 0.85 kg.

B. Dynamic Model

In the following analysis, the rotary elements of the compliant actuator are converted into equivalent translational elements, without loss of generality. Then, the soft linear spring and the torsional spring are referred to as the low-stiffness spring and the high-stiffness spring, respectively. The compliant actuator with two-stage stiffness is shown in Fig. 2 [20], where $m_1$ denotes the mass of the motor, $m_2$ denotes the mass of ball screw and spur gear, $k_1$ and $k_2$ are the high stiffness and the low stiffness, respectively, $x_1$, $x_2$, and $x_3$ denote the positions of the motor, the ball screw and the spur gear,
and the load respectively, \(b_m\), \(b_1\), and \(b_2\) denote the damping coefficients for the motor, the high-stiffness spring, and the low-stiffness spring, respectively, and \(u\) is the control input which is also the exerted force.

The dynamic equation for the compliant actuator can be described using Newton’s law as

\[
\begin{aligned}
m_1\ddot{x}_1 + b_m\dot{x}_1 + b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) &= u \quad (1) \\
m_2\ddot{x}_2 + b_2(\dot{x}_2 - \dot{x}_3) + k_2(x_2 - x_3) &= b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2). \quad (2)
\end{aligned}
\]

Equation (1) describes the dynamic model of the motor, while (2) describes the dynamic model of the ball screw and the spur gear. The two subsystems are linked by the forces \(k_1(x_1 - x_2) + b_1(\dot{x}_1 - \dot{x}_2)\).

In the compliant actuator, the low-stiffness spring works in a low-force range, and it is saturated when the force exerted on the spring reaches its limit. Then, the high-stiffness spring is activated to work in a high-force range. The different modes of the compliant actuator are summarized as follows.

1) **Low-Stiffness Mode:** When the low-stiffness spring plays a dominant role, the dynamic model can be as shown in Fig. 3. That is, the high-stiffness is not activated and thus \(x_1 \approx x_2\). The output force of the actuator is then obtained as \(F_o = k_2(x_2 - x_3) + b_2(\dot{x}_2 - \dot{x}_3)\) [20].

2) **High-Stiffness Mode:** When the high-stiffness spring plays a dominant role, the dynamic model is as shown in Fig. 4. That is, the low-stiffness spring is almost saturated such that \(|x_2 - x_3| \approx c\), where \(c\) is a positive constant. Hence, we have \(k_2|x_2 - x_3| \approx k_2c\), and \(b_2(\dot{x}_2 - \dot{x}_3) \approx 0\), and the output force is obtained as \(F_o = k_1(x_1 - x_3) + b_1(\dot{x}_1 - \dot{x}_3)\) [20].

Due to the hard switching of two springs, the overall stiffness of the compliant actuator is discontinuous, as shown in Fig. 5, which is specified as

\[
k_c(F_o) = \begin{cases} 
k_1, & |F_o| > k_2c \\
k_2, & |F_o| \leq k_2c. 
\end{cases}
\]

The control objective for the compliant actuator with a two-stage stiffness is to drive the state variable \(x_2\) or \(x_1\) to the desired time-varying trajectory \(x_d\), in the low-stiffness mode or the high-stiffness mode, respectively. Due to the discontinuity of the overall stiffness, finding a solution to the tracking control problem of the compliant actuator is nontrivial.

**Remark 1:** The ball screw used in our actuator has a high precision with a pitch of 2 mm/rev. Therefore, the actuator exhibits excellent reversibility, extremely low friction, and high efficiency, compared with conventional gearboxes. In this sense, the actuator is backdrivable even if the power is turned off. Also, in our design, the ball screw is fixed at both ends and only rotates to make the nut move in both directions. As the overall friction is very low, there is no noticeable difference in terms of friction and damping in the two directions. The backdrivability of the actuator has been tested in [26].

**Remark 2:** This paper considers the position control problem of the compliant actuator, where only the actuator dynamics in (1) and (2) is considered. When the compliant actuator is controlled to drive the robot joint, the load dynamics can be modeled as the well-known rigid robot dynamics [27], and the overall system is a high-order system consisting of both the robot dynamics and the actuator dynamics.

### III. Tracking Control Scheme

In this section, a continuous tracking controller is developed for the compliant actuator with two-stage stiffness, which is able to drive the two state variables, \(x_1\) and \(x_2\), to track the time-varying desired trajectory \(x_d\), with a smooth transition between the low-stiffness mode and the high-stiffness mode.
TABLE I

<table>
<thead>
<tr>
<th>Variations of Control Terms</th>
</tr>
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<tbody>
<tr>
<td>w(F_o)</td>
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<tr>
<td>Low-stiffness mode</td>
</tr>
<tr>
<td>High-stiffness mode</td>
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Two sliding variables are proposed for different modes of the actuator as

\[
\begin{align*}
    s_1 &= \dot{x}_1 - \ddot{x}_1 = \dot{x}_1 - \dot{x}_{o1} + a_1(x_1 - x_{o1}) \\
    s_2 &= \dot{x}_2 - \ddot{x}_2 = \dot{x}_2 - \dot{x}_{o2} + a_2(x_2 - x_{o2})
\end{align*}
\]

where

\[
\begin{align*}
    \dot{x}_1 &= \ddot{x}_{o1} - a_1(x_1 - x_{o1}) = \ddot{x}_{o1} - a_1 \Delta x_1 \\
    \dot{x}_2 &= \ddot{x}_{o2} - a_2(x_2 - x_{o2}) = \ddot{x}_{o2} - a_2 \Delta x_2
\end{align*}
\]

are auxiliary variables, \(a_1\) and \(a_2\) are positive constants, and \(\Delta x_1 = x_1 - x_{o1}\) and \(\Delta x_2 = x_2 - x_{o2}\). When the actuator works in the low-stiffness mode, \(x_{o1} = x_1\), \(x_{o2} = x_2\), and thus \(s_1 = 0\), \(s_2 \neq 0\). That is, the sliding variable \(s_2\) works, while \(s_1\) remains zero. When the actuator works in the high-stiffness mode, \(x_{o1} = x_d\), \(x_{o2} = x_2\), and thus \(s_1 \neq 0\), \(s_2 = 0\). That is, the sliding variable \(s_1\) works, while \(s_2\) reduces to zero. Therefore, the two sliding variables \(s_1\) and \(s_2\) are employed in different control modes, to drive the state variables \(x_1\) or \(x_2\) to the desired trajectory \(x_d\), respectively.

Using the sliding variables \(s_1\) and \(s_2\), the dynamic model described by (1) and (2) can be written as

\[
\begin{align*}
    m_1 \ddot{x}_1 + m_1 \dddot{x}_r + b_m \dot{x}_1 + b_m \dddot{x}_r + b_1 (\dot{x}_1 - \dddot{x}_2) &+ k_1 (x_1 - x_2) = u \\
    m_2 \dddot{x}_2 + m_2 \dddot{x}_r + b_2 \dddot{x}_2 + b_2 \dddot{x}_r - b_2 \dot{x}_3 + k_2 (x_2 - x_3) &+ b_1 (\dot{x}_1 - \ddot{x}_2) = k_1 (x_1 - x_2).
\end{align*}
\]

The tracking control scheme for the compliant actuator with two-stage stiffness is now proposed as

\[
\begin{align*}
    u &= -k_{11} s_1 - k_{22} s_2 - b_2 \dot{x}_3 + k_2 (x_2 - x_3) \\
        &+ m_1 \dddot{x}_r + b_m \dddot{x}_1 + m_2 \dddot{x}_r + b_2 \dddot{x}_2
\end{align*}
\]

where \(k_{11}\) and \(k_{22}\) are positive constants. The first two terms in (11) denote the sliding control in the high-stiffness mode and the low-stiffness mode, respectively, and the other terms denote the dynamic compensation. The key novelty of the proposed controller (11) is that the new reference variables \(x_{o1}\) and \(x_{o2}\) are employed instead of the conventional desired trajectories. The reference variables can be treated as weighted sums between the desired trajectory and the state variables. The feedback control is activated when the corresponding reference variable is specified as the desired trajectory, and it is suspended when the reference variable is just the state variable. The variations in the control terms in different modes are summarized in Table I.

Substituting the control input (11) into (9), the closed-loop equation is obtained as

\[
\begin{align*}
    m_1 \ddot{s}_1 + (b_m + b_{11}) s_1 - m_2 \ddot{s}_2 - b_2 \dddot{s}_2 + k_{22} s_2 + b_2 \dddot{s}_3 \\
        - k_2 (x_2 - x_3) + b_1 (\dot{x}_1 - \ddot{x}_2) + k_1 (x_1 - x_2) &= 0
\end{align*}
\]

First, a force region is formulated to monitor the variation of the output force as

\[
f(F_o) = F_o^2 - (k_2 c)^2 \leq 0.
\]

When \(f(F_o) \leq 0\), \(|F_o| \leq k_2 c\), the actuator works in the low-stiffness mode, and \(x_2\) is controlled to track \(x_d\). When \(f(F_o) > 0\), \(|F_o| > k_2 c\), the actuator works in the high-stiffness mode, and \(x_1\) is controlled to track \(x_d\).

Then, a weight factor \(w(F_o)\) is defined as

\[
w(F_o) = 1 - \frac{(\text{min}[0, \text{min}(0, f(F_o))])^4 - [(kk_2 c)^2 - (k_2 c)^2]^4}{[(kk_2 c)^2 - (k_2 c)^2]^4}
\]

where \(0 < \kappa < 1\) is a constant that determines the gradient of the weight between 0 and 1. The weight factor \(w(F_o)\) is second-order continuous, and an illustration of \(w(F_o)\) is shown in Fig. 6. The weight factor is 0 when the output force is outside the force region where \(f(F_o) > 0\), and it smoothly increases to 1 when the output force is inside the force region where \(f(F_o) \leq 0\). Note that the force \(F_o\) is obtained by measuring the displacements \(x_1\), \(x_2\), and \(x_3\), and thus the force sensor is not required.

Next, two reference variables \(x_{o1}\) and \(x_{o2}\) are specified as

\[
\begin{align*}
    x_{o1} &= (1 - w(F_o)) x_d + w(F_o) x_1 \\
    x_{o2} &= w(F_o) x_d + (1 - w(F_o)) x_2.
\end{align*}
\]

From (5) and (6), when the actuator works in the low-stiffness mode, \(w(F_o) = 1\). Then, the reference variables in (6) can be written as \(x_{o1} = x_1\), \(x_{o2} = x_2\). When the actuator works in the high-stiffness mode, \(w(F_o) = 0\). Then, the reference variables become \(x_{o1} = x_d\), \(x_{o2} = x_2\).
which also implies that
\[
b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = \\
-m_1\dot{s}_1 - (b_m + k_s)\dot{s}_1 + m_2\dot{x}_2 + b_2\dot{x}_2 \\
- k_{s_2}\dot{s}_2 - b_2\dot{x}_3 + k_2(x_2 - x_3).
\]
(13)
Substituting (13) into the right side of (10) yields
\[
m_1\dot{s}_1 + (b_m + k_s)\dot{s}_1 + m_2\dot{s}_2 + (b_2 + k_{s_2})s_2 = 0.
\]
(14)
The block diagram of the closed-loop system is shown in Fig. 7.

We can now state the following theorem.

**Theorem 1:** The proposed tracking controller (11) for the compliant actuator system with two-stage stiffness described by (14) gives rise to the convergence of tracking errors in the **low-stiffness mode** and the **high-stiffness mode**, respectively.

**Proof:** When the compliant actuator works in the **low-stiffness mode**, \(s_1 \to 0\), and (14) is simplified as
\[
m_2\dot{s}_2 + (b_2 + k_{s_2})s_2 = 0
\]
(15)
which implies that \(s_2 \to 0\). Since \(s_2 = \Delta \dot{x}_2 + \alpha_2 \Delta x_2\), the convergence of \(s_2 \to 0\) implies that \(\Delta \dot{x}_2 \to 0\) and \(\Delta x_2 \to 0\). Since \(\omega(F_o) = 1\) in the **low-stiffness mode**, \(s_{o_2} = \dot{x}_d\) from (6), and thus we have \(\dot{x}_2 \to \dot{x}_d\) and \(x_2 \to x_d\) as \(t \to \infty\). That is, the state variable \(x_2\) tracks the desired trajectory when the actuator works in **low-stiffness mode**.

When the compliant actuator works in the **high-stiffness mode**, \(s_2 \to 0\), and hence (14) is simplified as
\[
m_1\dot{s}_1 + (b_m + k_s)\dot{s}_1 = 0
\]
(16)
which then implies that \(s_1 \to 0\). Since \(s_1 = \Delta \dot{x}_1 + \alpha_1 \Delta x_1\), the convergence of \(s_1 \to 0\) implies that \(\Delta \dot{x}_1 \to 0\) and \(\Delta x_1 \to 0\). Since \(\omega(F_o) = 0\) in the **high-stiffness mode**, \(x_{o_1} = \dot{x}_d\) from (6), and thus we have \(\dot{x}_1 \to \dot{x}_d\) and \(x_1 \to x_d\) as \(t \to \infty\). That is, the state variable \(x_1\) tracks the desired trajectory when the actuator works in **high-stiffness mode**.

During the transition between the two control modes, we have the following theorem to state the convergence.

**Theorem 2:** The proposed tracking controller (11) guarantees the convergence of tracking errors when the compliant actuator transits between the **low-stiffness mode** and the **high-stiffness mode**, if the control gains \(\alpha_1, \alpha_2, k_{s_1}\), and \(k_{s_2}\) are chosen such that
\[
a_1 = a_2 = \alpha \\
k_{s_1} = k_{s_2} = k_s
\]
(17)
where \(k_s\) is set sufficiently large.

**Proof:** It has been shown in Theorem 1 that the control gains \(\alpha_1, \alpha_2, k_{s_1}\), and \(k_{s_2}\) can be chosen as any positive values to ensure the convergence of tracking errors, when the actuator works in either the **low-stiffness mode** or the **high-stiffness mode**. We now consider the transition stage where \(0 < \omega(F_o) < 1\). During the transition stage, the high-stiffness spring is not activated such that \(x_1 \approx x_2\). Then, from (6) and (7), we have
\[
s_1 = \dot{x}_1 - \dot{x}_d + \alpha_1(x_1 - x_{o_1}) \\
= (1 - \omega(F_o))(\dot{x}_1 - \dot{x}_d) \\
+ [\alpha_1(1 - \omega(F_o)) - \dot{\omega}(F_o)](x_1 - x_d)
\]
and
\[
s_2 = \dot{x}_2 - \dot{x}_d + \alpha_2(x_2 - x_{o_2}) \\
= \omega(F_o)(\dot{x}_2 - \dot{x}_d) + [\alpha_2\omega(F_o) + \dot{\omega}(F_o)](x_2 - x_d).
\]
Note that \(x_1 \approx x_2\), and also \(\alpha_1 = \alpha_2 = \alpha\) when condition (17) is satisfied. Adding (18) with (19) yields
\[
s_1 + s_2 = (\dot{x}_2 - \dot{x}_d) + \alpha(x_2 - x_d).
\]
(20)
Since \(k_{s_1} = k_{s_2} = k_s\), (14) can be written as
\[
m_{s_1}\dot{s}_1 + \left(\frac{b_m}{k_s} + 1\right)s_1 + m_{s_2}\dot{s}_2 + \left(\frac{b_2}{k_s} + 1\right)s_2 = 0.
\]
(21)
When \(k_s\) is chosen sufficiently large such that \((m_1/k_s), (b_m/k_s), (m_2/k_s), (b_2/k_s) \to 0\), (21) reduces to
\[
s_1 + s_2 = 0.
\]
(22)
From (20), it is obtained that
\[
(\dot{x}_2 - \dot{x}_d) + \alpha(x_2 - x_d) = 0
\]
(23)
which implies that \(x_2 \to x_d\), and \(\dot{x}_2 \to \dot{x}_d\) as \(t \to \infty\).
Remark 3: The choice of the control parameters should be balanced between the maximum motor input force and the tracking performance. Then, \( a_i \) and \( k_{si} \) \((i = 1, 2)\) are fine-tuned together to achieve the desired transient and steady-state error performance. Short rise time and smaller steady-state error can be achieved by tuning \( a_i \) up and better velocity tracking can be done by tuning \( k_{si} \) up.

IV. MODEL-BASED OBSERVER

Note that the acceleration information \( \ddot{x}_1 \) and \( \ddot{x}_2 \) is required in the derivatives of reference variables \( \dot{x}_{r1} \) and \( \dot{x}_{r2} \), which is thus required in the control input \( u \) (11). To eliminate the requirement of the acceleration information, observers are developed to estimate \( \dot{x}_{r1} \) and \( \dot{x}_{r2} \), respectively.

First, the dynamic model of motor is reproduced as
\[
m_1 \ddot{x}_1 + b_m \dot{x}_1 + b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = u. \tag{24}
\]
Then, an observer can be developed for the estimated reference variable \( \dot{x}_{r1} \) as
\[
\begin{align*}
\dot{x}_{r1} &= \eta_1 + \beta_1 \dot{e}_1 \\
\dot{\eta}_1 &= m_1^{-1}(u - (b_m + b_1)\eta_1 + b_1 \dot{x}_1 - k_1(x_1 - x_2) + k_{e1}\dot{e}_1)
\end{align*}
\tag{25}
\]
where \( \dot{x}_{r1} \) denotes the estimated signal of \( x_{r1} \), \( \dot{e}_1 = x_{r1} - \dot{x}_{r1} \) is the observation error, \( \eta_1 \) is an auxiliary variable, and \( \beta_1 \) and \( k_{e1} \) are positive constants. By following the similar development in [28], it can be shown that the observer (25) ensures \( e_1 \to 0 \). That is, \( \dot{x}_{r1} \to x_{r1} \) as \( t \to \infty \).

Then, the dynamic model of ball screw and spur gear is reproduced as
\[
m_2 \ddot{x}_2 + b_2(\dot{x}_2 - \dot{x}_3) + k_2(x_2 - x_3) = b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2). \tag{26}
\]
Then, another observer can be developed for the estimated reference variable \( \dot{x}_{r2} \) as
\[
\begin{align*}
\dot{x}_{r2} &= \eta_2 + \beta_2 \dot{e}_2 \\
\dot{\eta}_2 &= m_2^{-1}(-b_1(\dot{x}_2 - \dot{x}_3) + b_1 \dot{x}_1 + k_1(x_1 - x_2) - \dot{x}_3
\tag{27}
\end{align*}
\]
where \( \dot{x}_{r2} \) denotes the estimated signal of \( x_{r2} \), \( \dot{e}_2 = x_{r2} - \dot{x}_{r2} \) is the observation error, and \( \eta_2 \) is also an auxiliary variable, and \( \beta_2 \) and \( k_{e2} \) are positive constants. Similarly, it can be shown that \( \dot{x}_{r2} \to x_{r2} \) as \( t \to \infty \).

Using the estimated reference variables \( \dot{x}_{r1} \) and \( \dot{x}_{r2} \), the two sliding variables are revised as
\[
\begin{align*}
\hat{\xi}_1 &= \dot{x}_1 - \dot{x}_{r1} + a_1(x_1 - \dot{x}_{r1}) \\
\hat{\xi}_2 &= \dot{x}_2 - \dot{x}_{r2} + a_2(x_2 - \dot{x}_{r2}). \tag{28}
\end{align*}
\]
Therefore, the requirement of the acceleration information and its derivative is eliminated.

V. EXPERIMENTS

The proposed continuous tracking control scheme was implemented in the compliant actuator system with two-stage stiffness as shown in Fig. 8. The system consists of the actuator mounted on the testing jig, a real-time control system, a motor driver, and a host PC. The motor driver is Elmo Harmonica 5/60, which can provide a maximum power output of 200 W and a continuous output current of 5 A. The control algorithm is realized in the NI CompactRIO 9074 embedded control system (National Instruments), and the feedback information is obtained using the NI data acquisition system, which consists of NI 9215 (analog input module), NI 9263 (analog output module), and NI 9516 (encoder module). The sampling frequency is set as 2 kHz.

The switching point of the two springs shown in Fig. 5 is calibrated as \( F_0 = 240 \) N. Therefore, the low-stiffness spring works at \(-240 \leq F_0 \leq 240\) N, while the high-stiffness spring works at \( F_0 > 240 \) N, or \( F_0 < -240 \) N. In experiments, the position of load is fixed and calibrated as \( x_3 \equiv 0 \), for testing purposes. The dynamic parameters in (1) and (2) are obtained by referring to [20]. A series of time-varying desired trajectories were specified to illustrate the performance of the proposed controller in the low-stiffness mode, the high-stiffness mode, and during the transition between two modes.

The parameter \( \kappa \) in (5) was set as \( \kappa = 0.9 \). The control parameters in (11) were set as \( k_{e1} = k_{e2} = 100 \), and \( a_1 = a_2 = 0.06 \). Observers in (25) and (27) were developed to estimate the reference variables \( x_{r1} \) and \( x_{r2} \), where the control gains were set as \( k_{e1} = k_{e2} = 100 \) and \( \beta_1 = \beta_2 = 60 \).

In the first experiment, the desired trajectory was specified as \( x_d = 0.012 \sin(2\pi t) \) m. The compliant actuator worked in the low-stiffness mode, such that only the low-stiffness spring was activated and the weight factor \( w(F_0) \) remained 1. Therefore, the reference variables \( x_{r1} = x_1, x_{r2} = x_d \). The path of the position variable \( x_2 \) and the reference variable \( x_{r2} \) is shown in Fig. 9(a). The corresponding tracking error is shown in Fig. 9(b), which is less than 0.003 m. The observation error, i.e., \( x_{r2} - \dot{x}_{r2}, \) is shown in Fig. 9(c), which is around 0.002 m. The bounded control input \( u \) is shown in Fig. 9(d).

In the second experiment, the desired trajectory was specified as \( x_d = 0.08 + 0.012 \sin(2\pi t) \) m. The compliant actuator worked in the high-stiffness mode such that only the high-stiffness spring was activated and the weight factor \( w(F_0) \) remained zero. Therefore, the reference variables \( x_{r1} = x_d, x_{r2} = x_2 \). The path of the position variable \( x_1 \) and the reference variable \( x_{r1} \) is shown in Fig. 10(a). The corresponding tracking error is shown in Fig. 10(b), which is less than 0.005 m. The observation error, i.e., \( x_{r1} - \dot{x}_{r1}, \)
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Fig. 9. Experiment 1: Low-stiffness mode. (a) Path of $x_2$ and $x_{o2}$. (b) Tracking error $x_2 - x_{o2}$. (c) Observation error $x_{o2} - \hat{x}_{o2}$. (d) Control input $u$.

Fig. 10. Experiment 2: High-stiffness mode. (a) Path of $x_1$ and $x_{o1}$. (b) Tracking error $x_1 - x_{o1}$. (c) Observation error $x_{o1} - \hat{x}_{o1}$. (d) Control input $u$.

Fig. 11. Experiment 3: Transition between two modes. (a) Path of $x_2$ and $x_{o2}$. (b) Path of $x_1$ and $x_{o1}$. (c) Tracking error $x_2 - x_{o2}$. (d) Tracking error $x_1 - x_{o1}$. (e) Observation error $x_{o2} - \hat{x}_{o2}$. (f) Observation error $x_{o1} - \hat{x}_{o1}$; (g) control input $u$.

is shown in Fig. 10(c), which is around 0.001 m. The control input $u$ is shown in Fig. 10(d), which is also bounded.

In the third experiment, the desired force trajectory was specified as $x_d = 0.06 + 0.012\sin(2\pi t) \text{ m.}$ The compliant actuator transited between the low-stiffness mode and the high-stiffness mode. The path of the position variable $x_2$ and the reference variable $x_{o2}$ is shown in Fig. 11(a), and the path of the position variable $x_1$ and the reference variable $x_{o1}$ is shown in Fig. 11(b). It can be seen that when the actuator works in the low-stiffness mode, $x_{o1} = x_1$, and $x_{o2} = x_d$, i.e., $x_1$ is controlled to track itself while $x_2$ is controlled to track $x_d$. On the other hand, when the actuator works in the high-stiffness mode, $x_{o1} = x_d$, $x_{o2} = x_2$. Since the weight factor $w(F_o)$ is continuous, the reference variables are also continuous. The tracking error $x_2 - x_{o2}$ is shown in Fig. 11(c), which is less than 0.003 m, and the tracking error $x_1 - x_{o1}$ is shown in Fig. 11(d), which is less than 0.005 m. Note that the tracking error $x_2 - x_{o2}$ is larger when $w(F_o) = 1$ than those when $w(F_o) = 0$, which is because $x_2$ is controlled to track $x_d$ when $w(F_o) = 1$, and track itself when $w(F_o) = 0$. On the other hand, the tracking error $x_1 - x_{o1}$ is smaller when $w(F_o) = 1$ than those when $w(F_o) = 0$. The observation error $x_{o2} - \hat{x}_{o2}$ is shown in Fig. 11(e), which is around 0.001 m and the observation error $x_{o1} - \hat{x}_{o1}$ is shown in Fig. 11(f), which is around 0.002 m. The control input $u$ is shown in Fig. 11(g), which is also bounded. The experimental results show that the proposed control scheme has been successfully implemented in the compliant actuator with two-stage stiffness. Using the
proposed controller, the actuator is able to track the desired trajectory throughout the high-stiffness mode, the low-stiffness mode, and also the transition between two modes, with high tracking accuracy. The transition is smooth and stable, and the overall control input is continuous.

In the fourth experiment, a disturbance resulting from external forces on the actuator was added in the trajectory tracking, and the results are shown in Fig. 12. As seen in Fig. 12(b) and (d), the external disturbance, occurring at around $t \approx 3.8$ s, resulted in an increase of tracking error, but did not compromise the stability of the closed-loop system, i.e., the boundedness of tracking errors.

In the fifth experiment, the desired trajectory was specified as a ramp function with a frequency of 0.5 Hz, and the controller also transited between the high-stiffness mode and low-stiffness mode. The results are shown in Fig. 13, where the tracking errors are less than 0.006 m [see Fig. 13(c) and (d)], the observation errors [see Fig. 13(e) and (f)] are less than 0.01 m, and the control input was bounded [see Fig. 13(g)]. The results demonstrate the good
tracking performance of the proposed controller for different trajectories.

A comparison was also carried out between the proposed controller and the hard-switching controller. To implement the hard-switching controller, the weight factor \( w(F_o) \) was set as \( w(F_o) = 0 \) when \( F_o > 240 \) N, and \( w(F_o) = 1 \) when \( F_o \leq 240 \) N. The results are shown in Fig. 14, where the hard-switching controller results in a more significant chattering at the switching point, compared with the proposed continuous tracking controller.

VI. CONCLUSION

In this paper, a tracking control method has been proposed for the compliant actuator with two-stage stiffness. While the overall stiffness is discontinuous, the proposed control method ensures a continuous and automatic transition between the low-stiffness mode and the high-stiffness mode. The stability and convergence of the closed-loop system have been analyzed. The proposed control method has been successfully implemented in the compliant actuator system and the experimental results show that the actuator is able to track different time-varying trajectories in different modes. The proposed method can also be extended to various multiple feedback control schemes. Different feedback information can be integrated into a single continuous controller by constructing a weighted feedback term in a similar way. While each feedback is activated in a local range, the combination of multiple local feedback leads to the global convergence of the system to a desired objective. Therefore, the proposed method provides a general solution to many applications with embedded integration of behaviors.

REFERENCES


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