Analytical and numerical approaches to study echo laser pulse profile affected by target and atmospheric turbulence

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Abstract:
The integrated model of echo laser pulse profile (ELPP) of a target with arbitrary shape is studied under the condition of the ELPP affected by target and atmospheric turbulence simultaneously. The ELPPs of four typical targets (a plane, a cone, a sphere and an aspherical surface) are employed to test the validity of the model by analytical and numerical approaches. Based on simulations of the ELPP under different targets and atmospheric turbulence intensity, the results show a good agreement between two methods, and the ELPP of a target with discontinuous surface is more easily affected by atmospheric turbulence than that with a continuous surface. Besides that, we study the relationship between the number of grids and the relative error of analytical and numerical approaches, which are of interest to obtain the optimal number of grids used in the simulations.

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References and links
1. Introduction

With the fast development of pulse laser, many detecting sensors such as ranging sensors [1, 2] and three-dimensional (3D) sensors [3, 4] have been studied and developed. Moreover, target recognition based on pulse laser has attracted much attention these years due to its ability to acquire echo pulse with high accuracy and efficiency [5, 6]. Therefore, it has been widely used in applications including remote sensing [7], navigation [8] and tracking [9], etc. It is well known that echo laser pulse profile (ELPP) is one of the most important features, which only reflects the shape of target [9], but also affects the performances of 3D image [5].

For target recognition based on the ELPP, compared with traditional methods based on imaging process, the shape of the target is obtained by the use of the ELPP without optical scanning system [10]. Therefore, the method based on the ELPP is helpful to improve the efficiency of target recognition. For the 3D image, studying on the ELPP affected by atmospheric turbulence is significant to improve resolution and accuracy in practical applications [11]. To the best of our knowledge, previous studies have shown that the ELPP is mainly affected by the response of detectors [12], targets shape [13] and atmosphere, etc [14]. Although previous studies have presented relative results on the ELPP affected by targets [5, 15, 16], the results are based on the assumption of neglecting effect from atmospheric
turbulence. Meanwhile, the theoretical model of the ELPP of a target with arbitrary shape has not been studied under the condition of combinations of target and atmospheric turbulence. Actually, the ELPP is not only affected by target shape, but also affected by scintillation and fluctuation of atmosphere, especially for detecting the target at a long distance [11]. Besides that, the theoretical model of the ELPP is of interest to analyze practical applications based on laser profile through simulations, which is beneficial to improve design efficiency and reduce the cost of development.

In order to address these issues, we provide an analytical approach to study target recognition by the use of the ELPP combining target and atmospheric turbulence simultaneously, and the mathematic model of the ELPP of a target with arbitrary shape is deduced. To test model validity, the numerical approach is carried out by the use of classical FFT-based turbulent phase screen method [17, 18], which has been effectively verified to study beam propagation under the condition of atmospheric turbulence.

2. Method

2.1 Theoretical analysis

The principle schematic diagram is shown in Fig. 1. A target with arbitrary shape is illuminated by pulse laser beam. In terms of time-domain, the transmitting laser pulse beam is a function of Gaussian shape [19], which is written as

\[ P_t(t) = \frac{E_t}{\tau \sqrt{2\pi}} \exp\left(-\frac{t^2}{2\tau^2}\right), \]

where \( E_t \) is the original pulse energy, \( \tau \) is the transmitting pulse width. The target is sampled by \( M \times N \) small grids. Labeling the normal vector of the small grid \((\Delta x, \Delta y, \text{width} \times \text{length})\) as \( n \), so the theoretical power received from the small grid located at \((x, y)\) is written as [16, 20]

\[ P_r^{(\Delta x, \Delta y)} = \Delta x \Delta y I_0(x, y, r) P_r\left(t - 2\frac{z}{c}\right) \cos \theta_{(\Delta x, \Delta y)} \]

\[ = \Delta x \Delta y \frac{2}{\pi W^2_z} \exp(-2\frac{x^2 + y^2}{W^2_z}) P_r\left(t - 2\frac{z}{c}\right) \cos \theta_{(\Delta x, \Delta y)}, \]

where \( P_r(t) = \frac{E_t T_a T_o \eta_D P_t^{(\Delta x, \Delta y)}}{\sqrt{2\pi \tau_{rec}}} \exp\left[-\frac{1}{2\tau_{rec}^2}\left(t - 2\frac{R}{c}\right)^2\right], \cos \theta_{(\Delta x, \Delta y)} = \frac{1}{\sqrt{1 + z_x^2 + z_y^2}} \]

where \( I_0(x, y, r) \) is the propagated irradiance, \( z \) is the distance between the vertex of the target and the small grid. \( \theta_{(\Delta x, \Delta y)} \) is the angle between the normal vector \( n \) of the small grid and optical axis (z-axis), \( W_z \) is the beam radius after propagating \( R \) distance, \( c \) is the light speed, \( T_a \) is the one way atmospheric transmission, \( T_o \) is the receiver optics transmission efficiency, \( \eta_D \) is the quantum efficiency, \( R \) is the distance between the laser transmitter and the vertex of illuminated target, \( \rho_{(\Delta x, \Delta y)} \) is the reflectivity of the small grid, \( \tau_{rec} \) is the received echo pulse width, \( z_x \) is the partial differential of \( z \) relating \( x \), and \( z_y \) is the partial differential of \( z \) relating \( y \), which can be calculated by finite difference method.
If the target is tilted by rotating $\alpha$ degree clockwise along $y$-axis, the propagation distance between the transmitting laser and the small grid becomes $R' = R + z'$, and the angle $\theta_{(\Delta x, \Delta y)}$ between the normal vector $n$ of the small grid and the optical axis becomes $\theta'_{(\Delta x, \Delta y)}$. For a plane target, $z' = y \tan \alpha$ and $\theta'_{(\Delta x, \Delta y)} = \alpha$ when the plane is vertical to the optical axis before being tilted, i.e. $\theta_{(\Delta x, \Delta y)} = 0$ [16]. However, for a target with arbitrary shape, the coordinate of the small grid of the tilted target is written as

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Substituting Eq. (3) into Eq. (2), we can deduce the theoretical echo power from the small grid, i.e. the ELPP of the small grid, which is written as

$$p_{v}^{(\Delta x, \Delta y)} = \frac{2 \Delta x \Delta y}{\pi W_{g}^2} \exp \left[ - \frac{2 \left( x^2 + y^2 \right)}{W_{g}^2} \right] \frac{E_{0}^{2} T T \eta_{r} \rho_{\nu}(\Delta x, \Delta y)}{2 \pi r_{n}} \exp \left[ - \frac{1}{2 \pi r_{n}^2} \left( \frac{t - 2R}{c} \right)^2 \right] \cos \left( \theta'_{(\Delta x, \Delta y)} \right).$$

In the previous theoretical study, the target is viewed as a Lambert object [10], i.e., the reflectivity of the small grid $\rho_{\nu}(\Delta x, \Delta y)$ is viewed as a constant. However, it is not suitable for practical application, especially under the condition of the target is a rough target [21]. In order to keep the universality of the theory for practical use, bidirectional reflectivity distribution function (BRDF) is used to analyze the reflectivity of the small grid. Some typical methods are used to describe the BRDF, including Torrance-Sparrow [22], Blinn-Phong [23], Cook-Torrance [24], etc. Because the Torrance-Sparrow model is beneficial to describe the reflectivity of rough target, it is employed to calculate the reflectivity and its BRDF is written as

$$f_{v}(\theta, \theta', \phi_{r}) = \frac{k_{e} k_{s} \cos \beta}{1 + (k_{s}^2 - 1) \cos \beta} \exp \left[ b \left( 1 - \cos \gamma \right)^{a} \right] G(\theta, \theta', \phi_{r}) + \frac{k_{s}}{\cos \theta_{r}},$$

where $\theta$ is the angle between incident light and normal vector $n$ of the small grid, and is equal to $\theta'_{(\Delta x, \Delta y)}$, $\theta_{r}$ is the angle between reflective light and the normal vector $n$ of the small grid angle, $\phi_{r}$ is reflective azimuth angle. The triangle relationship of the BRDF is shown in Fig. 2. $k_{e}$, $k_{s}$, $\alpha$, $b$ are five parameters that vary from the different material of target. To
simplify the problems, a target with white diffuse reflector is taken as an example, whose five parameters are shown in Tab. 1. \( \delta_r, \delta_p, u_p, \) and \( v_p \) are the empirical parameters. \( \beta, \gamma \) are related to \( \theta_i, \theta_r, \phi, \) and \( G(\theta_i, \theta_r, \phi) \) is geometric attenuation. The relationships among those parameters are written as Eq. (6).

$$\cos^2 \gamma = 0.5(\cos \theta_i \cos \theta_r + \sin \theta_i \sin \theta_r \cos \phi + 1)$$

$$\cos \beta = \frac{\cos \theta_i + \cos \theta_r}{2 \cos \gamma}$$

$$G(\theta_i, \theta_r, \phi) = \frac{1 + \delta_r \tan \gamma_p}{(1 + w_p \tan^2 \gamma_p)(1 + w_p \tan^2 \gamma_p')}. $$

$$w_p = \delta_p \left(1 + \frac{u_p \sin \beta}{\sin \beta + v_p \cos \beta}\right) $$

$$\tan \theta'_p = \tan \theta_i \frac{\sin \theta_i + \sin \theta_r \cos \phi}{2 \sin \beta \cos \gamma}$$

$$\tan \theta''_p = \tan \theta_i \frac{\sin \theta_i + \sin \theta_r \cos \phi}{2 \sin \beta \cos \gamma}$$

$$\tan \gamma_p = \frac{\cos \theta - \cos \gamma}{2 \sin \beta \cos \gamma} $$

Combining Eq. (5) and Eq. (6), the reflectivity of the small grid can be expressed as

$$\rho_{r, 0}(\theta_i, \theta_r, 0) = f_r(\theta_i, \theta_r, 0) \cos \theta_r. $$
For the situation of detecting target at a long distance, the propagation distance $R$ is so large that little difference between $\theta_r$ and $\theta_i$ results in that reflective echo pulse signal cannot be detected by a receiving system [25]. For example, if the propagation distance $R$ is 10km, and the difference between $\theta_r$ and $\theta_i$ is only 0.1°, a receiving lens with 1.7m radius is needed to receive the reflective echo pulse signal, which increases the difficulty and cost of development. Therefore, it is reasonable to suppose that $\theta_r$ is equal to $\theta_i$.

The propagating distance from the laser transmitter to the small grid is $R' = R + z'$. Therefore, the increased beam radius $W_R'$ is written as [17, 26]

$$
\begin{align*}
\left\langle W_R^2 \right\rangle &= \left\langle W_i^2 \right\rangle (1 + 4q_c \Lambda_2)^2 \\
\left\langle W_i^2 \right\rangle &= W_0^2 + W_{\text{diff}}^2 + \left\langle W_{\text{turb}}^2 \right\rangle + W_{ab}^2 \\
W_{\text{diff}}^2 &= 4R^2 / k^2 W_0^2 \\
\left\langle W_{\text{turb}}^2 \right\rangle &= 2.18C_4^2 l_0^{-6} R^3 \\
W_{ab}^2 &= 8R^2 C_4^2 W_0^6
\end{align*}
$$

where $\langle \ast \rangle$ denotes the ensemble average of the parameter, $q_c$ is the non-dimensional roughness parameter, $\Lambda_2 = 2L / (k \langle W_i \rangle)$, $W_0$ is the initial beam radius of the laser beam, $k$ is the angular wavenumber of the light, i.e., $k = 2\pi / \lambda$, $l_0$ is the inner-scale size of turbulence, $C_4$ is the aberration strength of the non-ideal optical element.

We analyze pulse width from two aspects, i.e. target tilt and atmospheric turbulence. For the pulse broadening resulted from target tilt, which is written as [16]

$$
\left\langle \tau_{\text{tar}}^2 \right\rangle = \tau_0^2 + \frac{\tan^2 \left( \theta_{(\Delta r, \Delta \theta)} \right) \left\langle W_R^2 \right\rangle}{c^2},
$$

where $\tau_{\text{tar}}$ is the echo pulse broadening resulted from the target tilt, $\tau_0$ is the initial pulse width of the laser. Meanwhile, for the pulse width broadening resulted from atmospheric turbulence (from weak to strong turbulence intensity), it is written as [27, 28]

$$
\begin{align*}
\left\langle \tau_{\text{atm}}^2 \right\rangle &= \tau_0^2 + \frac{8 \times 0.3908 C_n^2 R L_0^{5/3}}{c^2} \\
L_0 &= \frac{5}{1 + \left( \frac{R - 7500}{2000} \right)^2},
\end{align*}
$$

where $C_n^2$ is the index of refraction structure constant, which represents the intensity of turbulence, and $L_0$ is the outer-scale size of turbulence. Therefore, the echo pulse width affected by these two aspects is written as

$$
\left\langle \tau_{\text{rec}}^2 \right\rangle = \left\langle \tau_{\text{tar}}^2 \right\rangle + \left\langle \tau_{\text{atm}}^2 \right\rangle.
$$

Based on the above discussions on the reflectivity, the beam radius, and the echo pulse width, we deduce the expression of the ELPP, which is the accumulation of the echo power of all small grids. Therefore, the ELPP can be described by the use of discrete and continues expressions, which are written as
Phase screen method to study beam radius

Phase screen method is an effective numerical approach to analyze laser propagation and imaging for atmospheric turbulence, which is based on the principle of statistics [18, 29]. The phase screen model is shown in Fig. 3. The beam of transmitting laser is a Gaussian beam, and the optical wave first experiences a Δz vacuum transmission and then arrives at the phase screen. After passing through N phase screens, the beam becomes distorted. Typical methods, such as fast Fourier transform (FFT) method [30], Zernike polynomials method [31] and covariance-based method [32] are used to generate turbulent phase screen. Compared with those methods, the FFT method is more efficient, smaller volume date processing, and is better suitable for generating an extra-large phase screen [33]. Therefore, we choose the FFT method to generate turbulent phase screen. The beam radius is written as [17]

\[
P_{mn} = \sum_{x_{mn}}^{2\Delta x_{mn}} 2\Delta x_{mn} \Delta y_{mn} \pi \left( \frac{W_{mn}^2}{W_{mn}^2 - \Delta y_{mn}^2} \right) \exp \left[ -\frac{2 \left( x_{mn}^2 + y_{mn}^2 \right)}{W_{mn}^2} \right] \cdot \exp \left[ -\frac{1}{2} \left( \frac{2 R}{c} \right) \right] \cdot \exp \left[ -\frac{1}{2} \left( \frac{2 R'}{c} \right) \right] \cos \left( \theta \right) \text{d}x \text{d}y.
\]

\[
P_{mn} = \int_{x}^{\infty} \int_{y}^{\infty} \frac{2 \left( x^2 + y^2 \right)}{\pi \left( W_{mn}^2 \right)} \cdot \exp \left[ -\frac{2 \left( x^2 + y^2 \right)}{W_{mn}^2} \right] \cdot \exp \left[ -\frac{1}{2} \left( \frac{2 R}{c} \right) \right] \cdot \exp \left[ -\frac{1}{2} \left( \frac{2 R'}{c} \right) \right] \cos \left( \theta \right) \text{d}x \text{d}y.
\]

(12)

where \( I (r, R') \) is the optical intensity distribution, and \( r = (x, y) \) is the transverse vector.

The high-frequency element of phase screen is written as
In order to avoid the under-sampling in the low spatial frequency of the FFT-based method, subharmonic method is used to compensate the low-frequency [34]. Supposing that the sub-harmonic level is \( p \), the low-frequency element of phase screen is written as

\[
\phi_{lf}(m,n) = \sum_{m'=1}^{N} \sum_{n'=1}^{N} h(m',n') f(m',n') \exp \left( \frac{2\pi i (m'N_m + n'N_n)}{N_m N_n} \right),
\]

where \( h(m',n') \) is the zero-mean unit-variance Hermitian complex Gaussian white noise process; \( f(m',n') \) is the discrete turbulence power spectrum; \( N_m \) and \( N_n \) are the sampling numbers along x-axis and y-axis respectively; \( m \) and \( n \) are the integer indices of phase screen; \( m' \) and \( n' \) are the integer indices; \( G_x \) and \( G_y \) are the sizes of phase screen; \( r_0 \) is the coherence length of the turbulence; \( f_x \) and \( f_y \) are the spatial frequencies, where \( f_x = m' \Delta f_x \) and \( f_y = n' \Delta f_y \), \( \Delta f_x = 1/G_x \) and \( \Delta f_y = 1/G_y \); \( p \) is the sub-harmonic level. The total phase screen is the sum of high-frequency element and low-frequency element, which is written as

\[
\phi(m,n) = \phi_{hf}(m,n) + \phi_{lf}(m,n).
\]

After passing through \( N \) phase screens, the envelope of electric field \( U(x, y, N \Delta z) \) can be obtained by the use of iteration [18], which is written as

\[
U(x, y, N \Delta z) = \exp \left[ -i \phi(m,n) \right] \text{FFT} \left[ B_x \cdot \text{FFT} \left[ U(x, y, (N-1) \Delta z) \right] \right],
\]

where \( B_x \) is the Fourier transformation of the vacuum transmission term. Substituting Eq. (17) into Eq. (13), the beam radius \( W_R \) after propagating \( R' \) distance, i.e. \( R' = N \Delta z \) can be obtained by Monte Carlo method, which is written as

\[
W_R^2 = \frac{2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dr^2 r^2 |U(x, y, R')|^2 \, dr \, dr}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dr^2 |U(x, y, R')|^2}. \tag{18}
\]

3. Simulations and results

3.1 Modeling verification

In order to verify the analytical and numerical approaches above, we compare our results with two previous studies [16, 17] under the same parameters respectively. The results of our simulation results are shown in Fig. 4. We can see that the echo pulse width and beam radius are two key parameters of the ELPP. Therefore, we compare our results with the two previous studies from the two aspects. First, from the aspect of the echo pulse width, we can find that our simulation results agree well with the results of [16]. Furthermore, the maximum relative error of \( \Delta t \), which is approximately equal to 2.35 times of the echo pulse width, between the two results is 0.9% \((|5.74-5.69|/5.69)\) when the tilt angle is 75°. Second, from the aspect of the
beam radius, we can see that the tendency of the beam radius under different propagation distance agrees with the results of [17], i.e., the beam radius increases with the increasing of propagation distance and turbulence intensity. Moreover, the difference values of the beam radius between two results under $C_n^2 = 2.43 \times 10^{-16}$ and $C_n^2 = 1 \times 10^{-15}$ at 5km propagation distance are 0.003m, and 0.004m, respectively. The maximum relative error is 6% ($|0.063-0.059|/0.059$) when $C_n^2 = 1 \times 10^{-15}$ at 5km propagation distance. From the two comparative results, we can see that our simulation results agree well with the two previous studies. It indicates that our simulations operate correctly.

3.2 Simulation parameters

According to the analysis above, we carry out the simulations combining effects of both target and atmospheric turbulence. In order to validate the mathematical model for an arbitrary shape, three typical targets [10] (a plane, a cone, a sphere) and a complicated target (an aspherical surface) are selected. The expression of an aspherical target is [35]

$$z = \frac{cS^2}{1+\left[1-(K+1)c^2S^2\right]^2} + A_4S^4 + A_2S^6 + A_3S^8 + A_4S^{10}, \quad (19)$$

where $S^2 = x^2 + y^2$, $K$ is the function of the eccentricity, $c = 1/r$, which $r$ is the radius of curvature at the vertex of the surface, $A_1$, $A_2$, $A_3$, and $A_4$ are the aspheric deformation constants. The Geometry models of the four targets are shown in Fig. 5, and the corresponding parameters of pulse laser [17, 20] and the geometry size of the four targets are set as in Tab. 2.

![Fig. 5. Geometry models of the four targets. From (a) to (d) are the four targets of a plane, a cone, a sphere and an aspherical surface, respectively.](image-url)
Table 2. Parameter values used in the simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Targets</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<td>Plane Length</td>
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<tr>
<td>Wavelength ($\lambda$)</td>
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<td>Plane Width</td>
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<tr>
<td>Initial pulse width ($\tau_0$)</td>
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<td>Cone Bottom radius</td>
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<tr>
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<td>Cone Length</td>
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<td>Sphere Curve radius</td>
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<tr>
<td>Quantum efficiency ($\eta_D$)</td>
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<td>Sphere Section radius</td>
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<tr>
<td>One way atmospheric transmission ($T_a$)</td>
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<td>$r$</td>
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<tr>
<td>Turbulence intensity($C_n^2$)</td>
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</table>

3.3 ELPPs vs tilt angle of target

For studying the effect of the tilt angle of the target, the turbulence intensity keeps unchanged ($C_n^2 = 3 \times 10^{-15}$, medium turbulence) and the tilt angle varies from $0^\circ$, $15^\circ$, $30^\circ$, $45^\circ$, and $60^\circ$ respectively. By comparing ELPPs of the four targets, shown in Fig. 6, we find that there are obvious differences in amplitude, pulse width, and peak offset of the ELPPs. (1) The amplitudes of the ELPPs of the four targets are different with the increase of the tilt angle. For example, the amplitude of the plane target decreases from 0.09W to 0.006W when $\alpha$ increases from $0^\circ$ to $60^\circ$. However, the amplitude of the cone target increases from 0.002W to 0.018W when $\alpha$ increases from $0^\circ$ to $60^\circ$, and the variation tendency of the amplitude agrees with that of [15]. (2) The pulse width is one of the remarkable characters of the ELPP, and we find that the pulse width of the ELPP of the targets (a plane and a sphere) increases with the increase of tilt angle. For example, for the target of the plane, the pulse width increases from 1.41ns to 9.12ns when $\alpha$ increases from $0^\circ$ to $60^\circ$, shown in Fig. 6(a). However, for the target of the cone and the aspherical surface, the widths decrease with the increase of tilt angle. For example, for the target of the aspherical surface, the pulse width decreases from 9.60ns to 5.38ns when $\alpha$ increases from $0^\circ$ to $60^\circ$, shown in Fig. 6(d). (3) The peak offsets of the ELPPs are also different with the different targets. For the plane target, with the increase of tilt angle from $0^\circ$ to $60^\circ$, the peak position of ELPP is unchanged, i.e. 0ns. However, for the target of the cone, the peak offset is 17ns (26ns $-9$ns = 17ns), shown in Fig. 6(b). For the targets of the sphere and the aspherical surface, the peak offsets are 5.3ns (−4.4ns$-0.8$ns = −5.3ns), and −4.8ns (−3.3ns$-1.5$ns = −4.8ns), respectively, shown in Fig. 6(c) and 6(d). The results show the target whose has the maximum offset is the cone target because of its discontinuous shape. Besides that, it is worth noting that the shapes of ELPPs of the four targets are also different. Under the condition of the tilt angle is $30^\circ$, the ELPP of the plane is a standard Gaussian profile. However, other three shapes of ELPP are Gaussian-like profiles. The peak ELPP of the cone is flat, and the ELPPs of the sphere and the aspherical surface tilt to the left. Therefore, it is beneficial to distinguish the shape of the target according to the analysis above.
3.4 ELPP affected by atmospheric turbulence

For studying the effect of atmospheric turbulence intensity, the tilt angle of the targets is constant ($\alpha = 15^\circ$) and the atmospheric turbulence intensity including weak atmospheric turbulence intensity ($C_n^2 = 1 \times 10^{-17}$), medium atmospheric turbulence intensity ($C_n^2 = 1 \times 10^{-15}$) and strong atmospheric turbulence intensity ($C_n^2 = 3 \times 10^{-13}$) are tested. The results of the ELPPs of the four targets under different turbulence intensity are shown in Fig. 7. Under the conditions of the weak and medium turbulence ($C_n^2 = 1 \times 10^{-17}$ and $C_n^2 = 1 \times 10^{-15}$), the amplitude of all ELPPs are affected not too much by the atmospheric turbulence intensity. For example, the amplitude of ELPP of the plane under the conditions of $C_n^2 = 1 \times 10^{-17}$ and $C_n^2 = 1 \times 10^{-15}$ are nearly identical, i.e. 0.086W, shown in Fig. 7(a). The amplitude of the ELPP of the cone decrease from 0.04W to 0.035W under the conditions of $C_n^2 = 1 \times 10^{-17}$ and $C_n^2 = 1 \times 10^{-15}$, respectively, shown in Fig. 7(b). However, when the target under the strong atmospheric turbulence intensity, the amplitudes of all ELPPs of the four targets decrease dramatically. For example, the amplitude of ELPP of the sphere decreases from 4.5mW to 2.3mW when $C_n^2$ increases from $1 \times 10^{-17}$ to $1 \times 10^{-13}$, shown in Fig. 7(c). Especially for the target of the aspherical surface, the amplitude of the ELPP decreases from 4.1mW to 0.2mW, shown in Fig. 7(d). A similar phenomenon can be found in Fig. 7(b) and 7(c), which shows that the target with complicated surface is easier affected by atmospheric turbulence intensity than the target with simple shape. Moreover, for the target with discontinuous shape, the position of the peak is affected by the strong atmospheric turbulence intensity. We can see the peak position of ELPP of the cone shifts from 0ns to 24ns, which show that the target with discontinuous shape is more easily affected by atmospheric turbulence intensity than the target with continuous shape. From the analysis above, we find that the targets with complicated and discontinuous surface are more easily affected by atmospheric turbulence than the target with continuous and simple shape.
3.5 Comprehensive comparison

As the analysis above, the ELPP is affected by the target and atmospheric turbulence simultaneously. Therefore, we carry out the comprehensive simulations based on analytical and numerical approaches. The parameters are set as follows: the distance from the target and the laser transmitting system is 10km, the tilt angle of the target is 15°, the turbulence intensity $C_n^2$ is $3 \times 10^{-13}$ (strong). It is well known that the real ELPP is affected by noise, such as background, dark current and shot noise, etc [20]. The distribution of noise are random [36] and conforms to Poisson distribution [25]. Therefore, the typical Poisson noise ($N \sim P(1 \times 10^{-4})$) is introduced in numerical approach, and the comparisons of ELPP between two methods are shown in Fig. 8, we can see that the ELPPs of all targets conform to the Gaussian distribution. To compare the two methods quantitatively, we evaluate the ELPPs from two aspects, namely the amplitude and the pulse width. First, from the amplitude aspect, the relative errors (REs) of the amplitude of the ELPPs between the two methods are 4.8% (0.050-0.049)/0.049 × 100%), 0.5% ((3.98-3.96)/3.96 × 100%), 1.7% ((2.29-2.25)/2.25 × 100%), and 5.8% ((2.18-2.06)/2.06 × 100%) for the plane, cone, sphere, and aspherical surface, respectively. Second, from the aspect of pulse width, the REs of the pulse width of the ELPPs between the two methods are 1.0% (|2.07-2.05|/2.05 × 100%), 1.7% (|28.3-28.8|/28.8 × 100%), 1.9% (|4.81-4.72|/4.72 × 100%), and 2.4% (|6.58-6.74|/6.74 × 100%) for the plane, cone, sphere, and aspherical surface, respectively. In terms of the whole tendency of ELPPs between the two methods, we draw the conclusion that the good agreement between the analytical method and the phase screen method validates our model of the ELPP.
4. Discussions

From the simulations above, we can see that ELPP is affected by target and atmosphere simultaneously. From the theoretical view, according to Eq. (12), besides the laser parameters, the ELPP is mainly determined by two key parameters, namely, the beam radius and the pulse width, and these two parameters are affected by the tilt angle of target and the atmospheric turbulence intensity. Therefore, based on the identical parameters of simulations above, we discuss these two parameters through the analytical and the numerical approaches.

4.1 Beam radius

From Eq. (8), we find that the beam radius is mainly determined by the propagation distance and the atmospheric turbulence intensity, and the corresponding results are shown in Fig. 9. We discuss the relationship between the beam radius and the propagation distance, turbulence intensity from two aspects. First, by fixing the propagation distance and varying the turbulence intensity, it is found that the beam radius increases with the increase of the atmospheric turbulent intensity. For example, under the condition of propagation distance at 10km, the beam radius increases from 0.12 m to 1.56 m when the turbulent intensity increases from $C_n^2 = 1 \times 10^{-17}$ to $C_n^2 = 3 \times 10^{-13}$. Second, the beam radius increases with the increase of the propagation distance when the turbulence intensity is a constant. For example, under the condition of $C_n^2 = 3 \times 10^{-13}$, the beam radius increases from 0.05 m to 8.09 m when the distance increases from 1km to 30km. The tendency of beam radius agrees with that of [17], which shows the correctness of our analytical approach. Figure 10(a) shows the relationship between the beam radius and the propagation distance based on phase screen method. The phase screen number and sampling number are set to be 200 and $512 \times 512$ respectively [18]. We find that the beam radius increases with the propagation distance and the tendency of Fig. 10(a) is identical to Fig. 9(b). Moreover, in order to quantitatively illustrate the difference
between analytical and numerical approaches, we evaluate the difference between the two methods by the use of relative error (RE) and maximum relative error (MRE) of the beam radius, i.e. \( |WR'_{\text{ana}}-WR'_{\text{num}}|/WR_{\text{ana}} \times 100\% \) and \( \max(|W_{\text{ana}}^{(i)}-W_{\text{num}}^{(i)}|/W_{\text{ana}}^{(i)}) \times 100\% \).

Under the condition of \( C_n^2 = 3 \times 10^{-13} \), we compare the difference between the two methods, shown in Fig. 10(b). The relative differences of beam radius between the two methods are 5.3% (\( (0.06-0.057)/0.057 \times 100\% \)), 1.9% (\( (1.59-1.56)/1.56 \times 100\% \)), and 0.25% (\( (8.07-8.09)/8.09 \times 100\% \)) under the propagation distances are 1 km, 10 km and 30 km, respectively. By comparing Fig. 9(b) and Fig. 10(a), the MREs of the two methods under different turbulence intensity are 1.7%, 2.5% and 5.3%, respectively. Therefore, the result illustrates that there is a good agreement between the two methods to calculate the beam radius.

**4.2 Echo pulse width**

The echo pulse width is another key parameter of the ELPP. According to Eqs. (9) and (10), in order to study the effect from tilt angle and turbulence intensity, we fix propagation distance, i.e., \( R = 10 \text{ km} \). Meanwhile, the echo pulse width is acquired by Gaussian fitting because different grid has its own echo pulse width and the ELPP conforms to Gaussian distribution. Here, we discuss the relationship between the echo pulse width and the propagation distance to illustrate the validity of the model, under the strongest turbulence intensity, i.e. \( C_n^2 = 3 \times 10^{-13} \). We use mean RE and MRE of echo pulse width to quantitatively evaluate the difference between the two methods. For the target of the plane, cone, sphere and aspherical surface, the mean REs of the echo pulse width between the two methods are 1.8%, 1.9%, 1.9%, and 1.9%, respectively.
2.5%, 3.6%, 3.8%. The MREs of pulse width of the four targets are 3.4% \((0.91-0.88)/0.88 \times 100\%\), 4.2% \(((1.552-1.489)/1.489 \times 100\%\)), 5.8% \((9.7-10.3)/10.3 \times 100\%\) and 6.3% \((6.40-6.02)/6.02 \times 100\%\), which are shown in Fig. 11, respectively. From the results of Fig. 11, we find the two methods agree well with each other, but it is worth noting that the MRE is related to the number of grids. For practical use, the optimal grids are important because too many grids cannot decrease MRE rather than increase computational cost. We find that the MRE of the four curves decreases with the increase of the number of grids. For example, for the target of the cone, the MRE decreases from 7.6% to 4.0% when the number of grids increases from 10 \(\times\) 10 to 200 \(\times\) 200. The details on MRE under different number grid can be seen in Fig. 12(b) and 12(c), and the fluctuation of Fig. 12(b) is obviously larger than Fig. 12(c). Furthermore, under the same MRE, the target with more complicated shape needs more grid number. For example, under the condition of the MRE is 5.2%, 10 \(\times\) 10 grids are used for the ELPP of the plane. However, 30 \(\times\) 30 grids are used for the ELPP of the cone. Besides that, the optimal number of grids is obtained from variation tendency of the MRE. For example, for the target of the aspherical surface, the difference of the MRE is 3.1% \((9.4\% - 6.3\%)\) when the number of grids increases from 10 \(\times\) 10 to 100 \(\times\) 100. However, the difference of MRE is 0.7% \((6.3\% - 5.6\%)\) when the number of grid increases from 100 \(\times\) 100 to 200 \(\times\) 200. Therefore, the optimal grid is 100 \(\times\) 100, which is used in the simulations above.

Fig. 11. Echo pulse width comparison of the analytical method and the phase screen method when \(C_n^2 = 3 \times 10^{-13}\). (a), (b), (c) and (d) are the plane, cone, sphere, and aspherical surface, respectively.
Fig. 12. The MRE echo pulse width comparison of the analytical method and the phase screen method when $C_n^2 = 3 \times 10^{-13}$. (a), (b), (c) and (d) are the plane, cone, sphere, and aspherical surface, respectively.

5. Conclusions

It is well known that the ELPP is affected by target and atmospheric turbulence simultaneously. However, the corresponding theoretical model has not been analyzed by previous studies. Here, we deduced the integrated model of the ELPP of a target with arbitrary shape under the combining effects of both target and atmospheric turbulence. The simulations of the four typical targets based on the analytical and numerical approaches are carried out, including the targets of a plane, a cone, a sphere and an aspherical surface. Some significant conclusions are achieved. (1) Based on the simulations of the ELPP under different targets and turbulence intensity, the comparative results show good agreement between analytical and numerical approaches, which shows the validity of model. (2) Based on different targets, the characters of ELPP are different, including the amplitude, the pulse width, and the offset of the peak position. First, under the parameters of simulations, the amplitude of ELPP of the plane is the highest while that of the aspherical surface is the lowest. Second, the pulse widths of ELPPs of the plane and sphere increase with the tilt angle, while the pulse widths of the cone and sphere decrease with the tilt angle. Third, the offset of the peak position is variant with different target. With the increase of the tilt angle, the peak position of ELPP of the plane is unchanged, and the offset of the cone is larger than that of the sphere and aspherical targets. (3) The ELPP of the target with discontinuous shape is easier affected by atmospheric turbulence than that of the target with continuous shape. (4) The accuracy of the model is determined by the number of grids. Based on the discussions on echo pulse width vs tilt angle of the target, the optimal number of grids is obtained to study the ELPP.

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