Control design of a novel compliant actuator for rehabilitation robots

Haoyong Yu\textsuperscript{a,b}, Sunan Huang\textsuperscript{a,n}, Gong Chen\textsuperscript{a,b}, Nitish Thakor\textsuperscript{a}

\textsuperscript{a}SiNAPSE, Singapore Institute of Neurotechnology, Singapore 117575, Singapore
\textsuperscript{b}Department of Bioengineering, National University of Singapore, Singapore 117575, Singapore

\textbf{A R T I C L E  I N F O}

Article history:
Received 1 October 2012
Accepted 23 August 2013
Available online 23 September 2013

Keywords:
Compliant actuator
Optimal control
Sliding mode
Smooth switching control

\textbf{A B S T R A C T}

Rehabilitation robots have direct physical interaction with human body. Ideally, actuators for rehabilitation robots should be compliant, force controllable, and back drivable due to safety and control considerations. Series Elastic Actuators (SEA) offers many advantages for these applications and various designs have been developed. However, current SEA designs face a common performance limitation due to the compromise on the spring stiffness selection. This paper presents a novel compact compliant force control actuator design for portable rehabilitation robots to overcome the performance limitations of current SEAs. Our design consists of a servomotor, a ball screw, a torsional spring between the motor and the ball screw, and a set of translational springs between the ball screw nut and the external load. The soft translational springs are used to handle the low force operation, while the torsional spring with high effective stiffness is used to deal with the large force operation. It is a challenging task to design the controller for such a novel design as the control system needs to handle both the force ranges. In this paper, we develop the force control strategy for this actuator. First, two dynamical models of the actuator are established based on different force ranges. Second, we propose an optimal control with friction compensation and disturbance rejection which is enhanced by a feedforward control for the low force range. The proposed optimal control with feedforward term is also extended to the high force range. Third, a switching control strategy is proposed to handle a transition between low force and high force control. The mathematical proof is given to ensure the stability of the closed-loop system under the proposed switching control. Finally, the proposed method is validated with experimental results on a prototype of the actuator system and is also verified with an ankle robot in walking experiments.

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\section{1. Introduction}

In classic industrial applications such as the autonomous welding systems for the automobile industry, robots are always designed with stiff actuators for precise and rapid position control with good repeatability. Stiff actuators are good for handling external disturbance forces and internal frictions, but cannot handle external impacts and shocks. Therefore, they are most suitable for working environments that are well defined with no direct physical interactions with humans. In recent years, due to the rapidly aging populations in most developed nations, there is a strong need for service robots, assistive and rehabilitation robots in both domestic [1,2] and hospital settings [3,4]. In these applications, the robots need to adjust with versatile unstructured environments as well as human demands [5]. Stiff actuators may be unsuitable for unstructured environments and the environments where human’s safety is a vital issue. This motivates the need of research on compliant actuators (CAs) because of their ability to safely interact with the user, and their ability to store and release energy in passive elastic elements. The most well-known CA is the series elastic actuator (SEA) [6,7] where a spring is placed between the motor and the load. Compared to a stiff actuator, series elastic actuators have the following benefits [8]:

- The actuators exhibit lower output impedance and backdriveability.
- Shock tolerance is greatly improved by the springs.
- The force transmission is smooth.
- Energy can be stored and released in the elastic element, thereby improving efficiency in applications.

Many different SEAs have been developed for rehabilitation robots to capitalize the advantages of the SEA concept. In [9,10], the SEA is designed based on a linear spring coupled to a ball screw which is connected to a dc motor. In [11,12], a Bowden cable is connected to linear spring to achieve a rotary SEA. In [13,14], rotary SEAs are designed in which torsional spring is used to transmit the output force. Although current SEAs have achieved reasonable
performance, they still face a common fundamental limitation, which is the fixed spring constant of the elastic element as discussed by Pratt et al. in [9, 15]. The performance of the SEAs largely depends on spring constant [15]. Soft spring produces high fidelity of force control, low output impedance, and reduces stiction, but also limits the force range and the force control bandwidth at high force range. On the other hand, stiff spring increases large force bandwidth, but reduces force fidelity. In order to achieve the desired output force/torque, most current SEAs are designed with very stiff springs, leading to compromised force control performance, low intrinsic compliance and back-drivability, and bulky and heavy systems. The novel design presented in this paper aims to overcome the above limitations of the conventional SEAs while improve the performance. This design concept was first proposed in [16,17] and supported with simulation results and simple experiments.

Apart from the mechanical development of SEAs, the control design of SEAs is also gaining attention in recent years. Many controllers have been presented for SEA and their performance have also been analyzed. The following literature will give a brief insight in this area. In [10, 11, 15, 18], pure PID control is used to produce a desired output force. In [9], PD plus feedforward control is used to improve the dynamical performance for a class of SEAs. In [13, 19–21], a type of cascaded control is presented to ensure stability in human interacting devices where a PI torque control is used in the outer loop, while a PI velocity control is used in the inner loop. In [6], a modified PID with feedforward term and human joint compensator is designed to generate desired force and low impedance. In [14], based on the model of a SEA, the authors further improve the human joint compensator used in [6] by adding a low pass filter. A disturbance observer is also designed to compensate the modeling error. In [22], the authors use a feedback plus feedforward force control which is enhanced by a disturbance observer to compensate for plant variations, where feedback and feedforward controls are optimally designed. However, all these current controllers are designed based on the actuators with fixed stiffness spring, either linear spring or torsional spring. For an actuator working with both types of springs in different force ranges, existing results are not available.

In this paper, we present a novel compliant actuator system and its force control design. Unlike existing SEAs, our system uses two types of springs (torsional and translational) at different force ranges. In order for this SEA to work well, there are challenges to be addressed in the control system. An adequate control strategy should be designed to deal with system dynamics and make a transition between high force and low force. The goal of the paper is to address these issues. First, two dynamical models are established with respect to low force range and high force range. Second, an optimal control is designed for low force range. Since there is a frictional behavior at the low force, we design a compensator in our controller to deal with the friction. As sliding mode control has been used as general tools for handling unknown nonlinear uncertainties [23], we incorporate it into our controller to deal with the unknown disturbance. The proposed optimal control plus feedforward term is also extended to high force range. Third, a switching control is proposed to make a smooth transition between low force and high force. The mathematical analysis is given to prove the stability of the closed-loop system. Finally, experimental results are provided to verify the effectiveness of the proposed method.

2. Compliant actuator design for gait rehabilitation robot

Gait disorders are common for patients post stroke and in most cases cannot be treated medically or surgically. Therefore, treatment often relies on rehabilitation service. Rehabilitation robotics has shown promise in providing patients with intensive therapy leading to functional gains. This involves the use of a robot exoskeleton device or end-effector device to help the patient retrain motor coordination by performing gait movement. Here, we are developing a portable wearable knee ankle robot for gait rehabilitation [17] based on the novel actuator design presented in this paper.

Fig. 1 shows the concept design of the robot. The modular system consists of an ankle foot module and a knee module. Each module is driven with the same compact compliant force controllable actuator as shown in Fig. 1. Based on the human lower limb biomechanics, it is known that the range of movement of the lower limb joints is within a known range during normal walking. We can make use of this property and use the simple rocker–slider mechanism to achieve a compact design that is most suitable for the exoskeleton. The core component in this exoskeleton is the compliant actuator as shown in Fig. 2.

Our novel actuator design consists of a servomotor with a rotary encoder, one torsional spring assembly with another rotary encoder, a pair of spur gear with appropriate gear ratio to transmit the motion to the ball screw which converts the rotary motion of the shaft to linear motion of the nut, a set of linear springs attached to the ball screw nut to transmit the force to a carriage which has a force output pin to transmit the force to the load (prospective robot link), and a linear position sensor installed in the carriage to measure the displacement of the linear spring. The two rotary encoders measure the angular deflection of the torsional spring.

In this design, the stiffness of the linear spring is chosen to provide the average targeted assistive torque, which is usually about one third of the peak torque. Therefore the linear spring is soft, small and light-weight. A very small torsional spring is used to achieve a very high effective spring constant at the output end because it is located in the high speed range. As we can see in the next section, the effective spring constant at the output is more than one hundred times that of the linear spring. Due to the difference in spring constant, when the actuator is working in the low force range, the force control is based on the linear spring and the torsional spring behaves like a rigid link. However, when the actuator is working in the high force range, the soft linear is fully compressed and the force control is based on the torsional spring. Therefore, we can achieve a much smaller physical size of the overall actuator compared to existing SEA designs and make it ideal for wearable exoskeleton application.

Fig. 3 shows actuator prototype built based on this design for an exoskeleton system for lower limb rehabilitation for stroke patients. The actuator is designed to be able to provide up to
60 Nm assistive torque at the lower limb joints. A Maxon DC brushless motor (EC 4-pole 120 Watt 36 V) is used for the design due to its lightweight (0.175 kg) and low moment of inertia. The ball screw selected from Eichenberger Gewinde AG has a pitch of 2 mm/rev and can output over 1000 N force. The linear springs have spring constant of 24 N/mm and a working stroke of 10 mm. They can provide an output force of 240 N before fully compressed. They are used to operate in the range of about 25% of the full force. The torsional spring has a spring constant of 0.29 Nm/rad and a deflection range of 72°. The incremental rotary encoder has a resolution of 1024 lines/rev. The total mass of the actuator is less than 0.85 kg.

3. Control system and controller design

Controlling a compliant force actuator can be defined as achieving output forces according to certain requirements. In this section, we describe a detailed control design for our novel actuator, including hardware configuration and model-based control algorithms.

3.1. Hardware of actuation system

To achieve the control objective, a closed-loop force control system must be designed. This involves sensor, driver and controller.

In our system, the force is measured according to Hooke's law:

\[ F = k \Delta x \]  

where \( k \) is spring constant and \( \Delta x \) is the displacement of a spring. This means that a spring with a displacement \( \Delta x \) generates a force \( F \). With this force–displacement relationship, we can use encoder sensor to determine the spring deflection and spring force. In the control system, two encoders are used: one encoder is placed in spur gear and the other is placed in dc motor. The former is used to measure the position of linear spring when the output load is fixed, while the difference between both encoders is used to measure the angular movement of the torsional spring. For the case where the output load is not fixed, one linear potentiometer which is placed in linear spring as shown in Fig. 2, is used to check the linear spring deflection.

The actuator is driven with a brushless dc motor. Three hall effect sensors built in the dc motor are connected to the driver and provide commutation information which are used to identify the position of the rotor. The commutators and current control maintain the optimum torque angle.

The controller is the main part of the control system. In our system, a NI CompactRIO 9074 embedded control and data acquisition system and a personal computer (PC) are used to implement two force range control laws.

3.2. Modeling of compliant actuator system

In order to analyze the actuator performance at the output end, which produces linear output force, the actuator is modeled as a system consists of only translational elements by converting the rotary elements to equivalent translational elements. The actuator model for the equivalent translational motion is shown in Fig. 4(a). In this model, \( F \) is motor input force, \( m \) is equivalent mass of the motor plus the torsional spring coupler and the encoder as derived in (2) where \( J_1 \) refers to moment of inertia of the motor and the torsional spring coupler and the encoder, \( p \) is the pitch of the ball screw, \( m_2 \) is equivalent mass of the ball screw and the gears as derived in (3) where \( J_2 \) refers to moment of inertia for the ball screw and the gears, \( k_{t1} \) is considered as the equivalent translational spring constant of the torsional spring \( k_t \) as derived in (4), \( b_1 \) and \( b_2 \) are the viscous damping for motor and ball screw respectively, and \( F_2 \) is output force.

\[ \begin{align*}
F &= k \Delta x \\
J_1 &= \text{moment of inertia of motor and torsional spring coupler and encoder} \\
J_2 &= \text{moment of inertia for ball screw and gears} \\
k_{t1} &= \text{translational spring constant of torsional spring} \\
b_1 &= \text{viscous damping for motor} \\
b_2 &= \text{viscous damping for ball screw} \\
F_2 &= \text{output force}
\end{align*} \]
\[ m_1 = f_1(2\pi/p)^2 \]  
\[ m_2 = f_2(2\pi/p)^2 \]  
\[ k_1 = k_2(2\pi/p)^2 \]  

The physical parameters of the actuator prototype are listed in Table 1, where both the original number and the equivalent translational values are given for the dynamic modeling and analysis purpose. We can see that the spring constant \( k_1 \) derived from the torsional spring is about 120 times that of \( k_2 \), meaning the torsional spring can be considered to be a rigid link when the output force is low and the actuator behaves like a normal SEA with the linear spring only. The model of the actuator can be simplified as shown in Fig. 4(b).

Based on the working force range and the assumptions made above we derive those mechanical models.

### 3.2.1. Modeling for low force case

Referring to Fig. 4(b), the equation of motion according to Newton’s second law is given by

\[ (m_1 + m_2)x_1 = F - k_2(x_1 - x_3) - b_2(x_1 - x_3) - b_ux_1 \]  

We can get the system force output by applying the Hook’s law, i.e., \( F_1 = k_2(x_2 - x_3) \). Assuming a fixed load end, \( x_3 = 0 \), the Eq. (5) can be written as

\[ F_1 = \frac{k_2}{m_1 + m_2} F - \frac{k_2}{m_1 + m_2} F_1 \]  
\[ \hat{F}_1 = \frac{k_2}{m_1 + m_2} F - \frac{b_2}{m_1 + m_2} \hat{x}_1 + \frac{b_u}{m_1 + m_2} \hat{x}_1 + |\Delta f(\hat{x}_1) + d_1| \]  

Table 1

<table>
<thead>
<tr>
<th>Parameters of the actuator prototype.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values of the hardware parameters for rotational motion</td>
</tr>
<tr>
<td>( J_1 = 5.517 \times 10^{-3} \text{ kg m}^2 )</td>
</tr>
<tr>
<td>( J_2 = 7.406 \times 10^{-3} \text{ kg m}^2 )</td>
</tr>
<tr>
<td>Torsional spring ( k_2 = 0.29 \text{ Nm/rad} )</td>
</tr>
<tr>
<td>Linear spring ( k_2 = 24 \times 10^3 \text{ N/m} )</td>
</tr>
<tr>
<td>Pitch of the ball screw ( p = 2 \times 10^{-3} \text{ m} )</td>
</tr>
<tr>
<td>Rotary encoder resolution = 1024/rev</td>
</tr>
<tr>
<td>Linear potentiometer 25 mm</td>
</tr>
<tr>
<td>Total weight of actuator 0.84 Kg</td>
</tr>
</tbody>
</table>

\[ \text{where } c \text{ is the same as in (7).} \]

### 3.3. Controller design

#### 3.3.1. Low force control

We first design the model-based feedback control. Define the force error \( e_1(t) = F_d(t) - F_1(t) \) and the derivative error is given by \( e_2 = e_1(t) = F_d(t) - F_1(t) \). Thus, we define a state being \( \mathbf{e} = [e_1, e_2] \) and the state equation is given by

\[ \dot{\mathbf{e}} = A_1 \mathbf{e} - B_1 u + B_1 \left[ \frac{m_1 + m_2}{c k_2} F_d + \frac{b_2 + b_u}{m_1} F_d + \frac{1}{c} F_d + \Delta f(\hat{x}_1) + d_1 \right] \]  

with

\[ A_1 = \begin{bmatrix} 0 & 1 \\ -k_2/m_1 & -b_2/m_1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ c k_2/m_1 \end{bmatrix} \]

and \( \Delta f(\hat{x}_1) = \mu_1 \text{sgn}(\hat{x}_1) \).

Consider the state \( [e_1, e_2]^T \). It is natural to think that a proportional-derivative (PD) controller should be employed, that is

\[ u(t) = K_p (F_d - F_1) + K_d (F_d - F_1) \]

where \( K_p \) and \( K_d \) are the PD parameters which should be chosen appropriately. Since the dominant linear model (without nonlinear and uncertain terms) is given by

\[ \mathbf{e} = A \mathbf{e} - B u \]

we will design the PD control based on this model. Substituting the control into the above equation yields

\[ \dot{\mathbf{e}} = \begin{bmatrix} 0 \\ -k_2/m_1 + c k_2 \end{bmatrix} K_p - \frac{b_2 + b_u}{m_1 + m_2} K_d \]

Since the characteristic equation of the closed-loop system is

\[ s^2 + \left( \frac{b_2 + b_u}{m_1 + m_2} + \frac{c k_2}{m_1} K_p \right) s + \frac{k_2}{m_1 + m_2} + \frac{c k_2}{m_1 + m_2} K_d \]

the pole assignment method can be used to design the PD parameters.

A more quantitative method to select PD parameters is to apply the Linear Quadratic Regulator (LQR) design theory. The LQR performance index is

\[ J = \int_0^\infty \mathbf{e}^T Q \mathbf{e} + \rho_1 u^2 \]

where \( Q_3 \) is a symmetric positive-definite matrix, and \( \rho_1 \) is a positive constant. The state feedback controller for the criteria (17) is

\[ u = \rho_1^{-1} \mathbf{e}^T P_1 \mathbf{e}(t), \]
where $P_1 = \begin{bmatrix} P_{11}^{(1)} & P_{12}^{(1)} \\ P_{21}^{(1)} & P_{22}^{(1)} \end{bmatrix}$ > 0 is the unique positive definite solution to the following equation

$$A_1^T P_1 + P_1 A_1 - \rho_1^{-1} P_1 B_1^T P_1 + Q_1 = 0$$  \hspace{1cm} (19)

with $Q_1 = \text{diag}(q_1^{(1)}, q_2^{(1)})$.

Note that the uncertainties such as nonlinear friction and disturbance are not considered in the above controller design. Next, the design of the sliding mode for the uncertain part of the system (11) will be discussed. Although the term $A_1(\dot{x}_1)$ is nonlinear, it can be estimated by using an off-line experiment. Thus, a relay mode is used to compensate the friction

$$u_r = \mu_1 \text{sgn}(\dot{x}_1)$$  \hspace{1cm} (20)

where $\mu_1$ should be estimated before the control design. For the term $d_1$, it is uncertain but it is assumed to be bounded by a value, $|d_1| \leq \sigma_1$  \hspace{1cm} (21)

Thus, a sliding mode control is used to reject the disturbance

$$u_s = c_1 \text{sgn}(\text{e}^T P_1 \dot{x}_1)$$  \hspace{1cm} (22)

where $c_1$ should be chosen to meet the conditions $c_1 > \sigma_1$.

Combining (18) with (22), the proposed low force control is given by

$$u = \rho_1^{-1} B_1^T P_1 \text{e}(t) + \mu_1 \text{sgn}(\dot{x}_1) + c_1 \text{sgn}((\text{e}^T P_1 \dot{x}_1) + u_g)$$  \hspace{1cm} (23)

with $u_g = \frac{m_1}{c_k} F_d + \frac{b_1 + b_2}{c_k} F_d + \frac{1}{c} F_d$.

**Theorem 1.** Consider the system (11). If the proposed low force controller (23) is applied to the system (11) and the condition $c_1 > \sigma_1$ is satisfied, then the state $\text{e}$ is asymptotically stable.

**Proof 1.** Consider a Lyapunov function

$$V = \text{e}^T P_1 \text{e}$$  \hspace{1cm} (24)

Its time derivative is given by

$$\dot{V} = \text{e}^T (A_1^T P_1 + P_1 A_1 - 2\text{e}^T P_1 B_1 u + 2\text{e}^T P_1 B_1 \sigma_1 + d_1)$$

$$\leq \text{e}^T (A_1^T P_1 + P_1 A_1 - 2\text{e}^T P_1 B_1 \rho_1^{-1} P_1 B_1^T P_1 \text{e} - 2\text{e}^T P_1 B_1 \mu_1 \text{sgn}(\dot{x}_1)$$

$$+ 2\text{e}^T P_1 B_1 (c_1 + 2\text{e}^T P_1 B_1 \text{e})$$

$$- 2\text{e}^T P_1 B_1 \sigma_1$$

$$- 2\text{e}^T P_1 B_1 \text{e}$$

$$\dot{V} \leq -2\text{e}^T (Q_1 + \rho_1^{-1} P_1 B_1^T P_1 \text{e} - 2\text{e}^T P_1 B_1 (c_1 - \sigma_1))$$

(25)

This implies that $\dot{V} < 0$ if $c_1 > \sigma_1$. The proof of Theorem 1 is completed. $\Box$

Next, let us see the high force control.

### 3.3.2. High force control

The controller is proposed according to the design method as presented in previous subsection.

Define the error $e_1(t) = F_d(t) - F_s(t)$ and the derivative error $e_2(t) = \dot{e}_1(t)$. Let $e(t) = [e_1(t), e_2(t)]^T$. The following error equation is obtained

$$\dot{e} = A_2 e - B_2 u + B_2 \left( \frac{m_1}{c_k} F_d + \frac{b_1 + b_2}{c_k} F_d + \frac{1}{c} F_d \right)$$  \hspace{1cm} (26)

$$A_2 = \begin{bmatrix} 0 & 1 \\ -\frac{b_1 + b_2}{m_1} & -\frac{1}{m_1} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ \frac{c_k}{m_1} \end{bmatrix}$$  \hspace{1cm} (27)

The LQR theory is used to design the feedback control. The linear model in the above system is

$$\dot{e} = A_2 e + B_2 u$$

The performance index for the LQR theory is given by

$$J = \int_0^\infty (e^T Q e + \rho_2 u^2) dt$$

(29)

where $Q$ is a symmetric positive-definite matrix, and $\rho_2$ is a positive constant. The feedback control is given below.

$$u = \rho_2^{-1} B_2^T P_1 \text{e}(t)$$

(30)

where $P_1 = \begin{bmatrix} P_{11}^{(2)} & P_{12}^{(2)} \\ P_{21}^{(2)} & P_{22}^{(2)} \end{bmatrix}$ > 0 is the positive definite solution to the following equation

$$A_1^T P_1 + P_1 A_1 - \rho_1^{-1} P_1 B_1^T P_1 + Q_2 = 0$$

(31)

with $Q_2 = \text{diag}(q_1^{(2)}, q_2^{(2)})$. Note that the LQR design can guarantee the stability of the closed-loop system.

For the other terms in the system (26), the feedforward control is used to compensate them.

$$u_{\text{ff}} = \frac{m_1}{c_k} F_d + \frac{b_1 + b_2}{c_k} F_d + \frac{1}{c} F_d$$

(32)

Finally, the proposed composite high force control law is given by

$$u = \rho_2^{-1} B_2^T P_1 \text{e}(t) + u_{\text{ff}}$$

(33)

Taking a similar proof as in Theorem 1, we have the following stability result for the high force.

**Theorem 2.** Consider the system (26). If the proposed high force controller (33) is applied to the system (26), then the state $\text{e}$ is asymptotically stable.

### 3.3.3. Switching control

Whenever the state of the system goes from low force range to high force range or from high force range to low force range, the control law should also change accordingly. The switching law is designed according to the mechanism model as shown in Fig. 4. It can be seen that the force control is naturally switched to the high force when the force becoming greater than the maximum value of the low force range, while it is switched to the low force when the force becoming less than the maximum value of the low force range. Based on these principles of the switching, we develop the following control law which results in a switching controller for the compliant actuator

$$u = \begin{cases} u_{\text{hf}}, & F_d \geq F_s \\ u_{\text{lf}}, & F_d < F_s \end{cases}$$

(34)

where $u_{\text{hf}}$ and $u_{\text{lf}}$ are the individual control for the high force or the low force control as described in previous sections, $F_d$ is the desired force, and $F_s$ is the switching point which is the maximum value of the low force range.

Theoretically speaking, the force transition control from the low force $F_l$ to the high force $F_h$ (or from the high force $F_h$ to the low force $F_l$) should be seamlessly switched at the switching point $F_s$ and executed on the corresponding mechanism system. Unfortunately, due to control error, such transition is sometime not
perfect. For example, when executing the low force control $u_l$, it may act at the mechanism system which belongs to the high force range; or the high force control $u_H$ executed may work at the mechanism system which belongs to the low force range. If such situations occur, a theoretical issue then arises as to what about the stability of the closed-loop system under the proposed switching control? It is quite important to respond to this question for the safety reason. The following theorem is given to establish a stability result for the switching control.

**Theorem 3.** Consider both the systems 11, 26. If the proposed switching controller (34) is applied to the system (11), or the system (26) and the following inequalities hold

$$
\bar{Q}_1 = Q_1 + 2\rho_1^2 P_1 B_1 P_1^T > 0
$$

(35)

$$
\bar{Q}_2 = 2\rho_2^2 P_2 B_2 P_2^T > 0
$$

(36)

then all the states $\mathbf{e}$ are uniformly bounded.

**Proof 2.** When applying the switching law (34) to the system, this results in four cases: Case 1 – when $F_x < F_d$, the control $u_l$ acts at the mechanism system which belongs to the low force range; Case 2 – when $F_x \geq F_d$, the control $u_H$ acts at the mechanism system which belongs to the high force range; Case 3 – when $F_x < F_d$, the control $u_l$ acts at the mechanism system which belongs to the high force range. For the cases 1 and 2, the stability of the closed-loop system can be guaranteed by Theorems 1 and 2, respectively. Here, we consider the stability issue under the cases 3 and 4.

Case 3. Consider the case where the switching control (34) is applied to the actuator which is at the low force range. Recalling the model (26) of the high force range, we have

$$
\mathbf{e} = A_{\mathbf{e}} \mathbf{e} - B_{\mathbf{e}} u + B_2 \left( \frac{m_1}{c_k} F_d + \frac{b_1 + b_m}{c_k} F_d + \frac{1}{c} F_d \right)
$$

(37)

The switching control takes the form given by

$$
u_l = \rho_1^{-1} P_1 \mathbf{e} + \mu_1 \text{sgn}(x_1) + c_1 \text{sgn}(\mathbf{e}^T P_1 \mathbf{e}) + u_{\mathbf{e}}
$$

(38)

Consider the Lyapunov function given by

$$
V = \mathbf{e}^T P_2 \mathbf{e}
$$

(39)

Its time derivative along the dynamics (26) with the switching control is given by

$$
\dot{V} = \mathbf{e}^T A_{\mathbf{e}} P_2^{\text{T}} + P_2 A_{\mathbf{e}} \mathbf{e} - 2 \rho_1 \mathbf{e}^T P_1 B_1 P_2^T \mathbf{e}
$$

$$
= 2 \mathbf{e}^T P_2 B_2 \mu_1 \text{sgn}(x_1) + c_1 \text{sgn}(\mathbf{e}^T P_1 \mathbf{e})
$$

$$
+ 2 \mathbf{e}^T P_2 B_2 \left[ \frac{m_1}{c_k} F_d + \frac{b_1 + b_m}{c_k} F_d + \frac{1}{c} F_d \right]
$$

$$
= -\mathbf{e}^T Q_2 \mathbf{e} - 2 \rho_2 \mathbf{e}^T P_2 B_2 P_2^T \mathbf{e}
$$

$$
- 2 \mathbf{e}^T P_2 B_2 \mu_1 \text{sgn}(x_1) + c_1 \text{sgn}(\mathbf{e}^T P_1 \mathbf{e})
$$

$$
+ 2 \mathbf{e}^T P_2 B_2 \left[ \frac{m_1}{c_k} F_d + \frac{b_1 + b_m}{c_k} F_d + \frac{1}{c} F_d \right]
$$

(40)

where we have used the Eq. (31). Since the inequality (36) holds, it follows that

$$
\dot{V} \leq -\lambda_{\min}(Q_2) \| \mathbf{e} \|^2 + 2 \| \mathbf{e} \| \| P_2 B_2 \mu_1 \text{sgn}(x_1) + c_1 \text{sgn}(\mathbf{e}^T P_1 \mathbf{e})
$$

$$
+ 2 \| \mathbf{e} \| \| P_2 B_2 \| \mu_1 + c_1 + \left| \frac{m_1}{c_k} F_d + \frac{b_1 + b_m}{c_k} F_d + \frac{1}{c} F_d \right|
$$

$$
\leq -\lambda_{\min}(Q_2) \| \mathbf{e} \|^2 + 2 \| \mathbf{e} \| \| P_2 B_2 \| \left( \mu_1 + c_1 + \left| \frac{m_1}{c_k} F_d + \frac{b_1 + b_m}{c_k} F_d + \frac{1}{c} F_d \right| \right)
$$

where $\lambda_{\min}(Q_1) = \min(\text{eigenvalues}(Q_1))$. It follows that $\dot{V} < 0$ as long as

$$
\| \mathbf{e} \| > 2 \| P_2 B_2 \| \left( \mu_1 + c_1 + \left| \frac{m_1}{c_k} F_d + \frac{b_1 + b_m}{c_k} F_d + \frac{1}{c} F_d \right| \right)
$$

(41)

Then, $\dot{V}$ is negative outside the compact set. This demonstrates that the state $\mathbf{e}$ is uniformly bounded at the high force range.

Case 4. Consider the case where the switching control (34) is applied to the actuator which is at the low force range. Recalling the model (11) of the low force range, we have

$$
\mathbf{e} = A_1 \mathbf{e} - B_1 u + B_1 \left[ \frac{m_1 + m_2}{c_k} F_d + \frac{b_2 + b_m}{c_k} F_d + \frac{1}{c} F_d + \Delta f(x_1) + d_1 \right]
$$

(42)

The switching control takes the form given by

$$
u_l = \rho_2^{-1} P_1 B_1 \mathbf{e} + u_{\mathbf{e}}
$$

(43)

Consider the Lyapunov function given by (24). Its time derivative along the dynamics (11) with the switching control is given by

$$
\dot{V} \leq \mathbf{e}^T A_{\mathbf{e}} P_1 + P_1 A_{\mathbf{e}} \mathbf{e} - 2 \rho_2 \mathbf{e}^T P_1 B_1 B_2 P_2 \mathbf{e}
$$

$$
+ 2 \mathbf{e}^T P_1 B_1 \left[ \mu_1 \text{sgn}(x_1) + d_1 \right]
$$

$$
+ 2 \mathbf{e}^T P_1 B_1 \left[ \frac{m_1}{c_k} F_d + \frac{b_1 + b_m}{c_k} F_d + \frac{1}{c} F_d \right]
$$

$$
= -\mathbf{e}^T Q_1 \mathbf{e} - 2 \rho_2 \mathbf{e}^T P_2 B_2 P_2^T \mathbf{e}
$$

$$
- 2 \mathbf{e}^T P_2 B_2 \mu_1 \text{sgn}(x_1) + c_1 \text{sgn}(\mathbf{e}^T P_1 \mathbf{e})
$$

$$
+ 2 \mathbf{e}^T P_2 B_2 \left[ \frac{m_1}{c_k} F_d + \frac{b_2 + b_m}{c_k} F_d + \frac{1}{c} F_d \right]
$$

(45)

where we have used the result of Theorem 1. Since the inequality (35) holds, it follows that

$$
\dot{V} \leq -\lambda_{\min}(Q_1) \| \mathbf{e} \|^2 + 2 \| \mathbf{e} \| \| P_2 B_2 \| \mu_1 \text{sgn}(x_1) + c_1 \text{sgn}(\mathbf{e}^T P_1 \mathbf{e})
$$

$$
+ 2 \| \mathbf{e} \| \| P_2 B_2 \| \left( \mu_1 + c_1 + \left| \frac{m_1}{c_k} F_d + \frac{b_2 + b_m}{c_k} F_d + \frac{1}{c} F_d \right| \right)
$$

$$
\leq -\lambda_{\min}(Q_1) \| \mathbf{e} \|^2 + 2 \| \mathbf{e} \| \| P_2 B_2 \| \left( \mu_1 + c_1 + \left| \frac{m_1}{c_k} F_d + \frac{b_2 + b_m}{c_k} F_d + \frac{1}{c} F_d \right| \right)
$$

(46)

where $\lambda_{\min}(Q_1) = \min(\text{eigenvalues}(Q_1))$. It follows that $\dot{V} < 0$ as long as

$$
\| \mathbf{e} \| > 2 \| P_2 B_2 \| \left( \mu_1 + c_1 + \left| \frac{m_1}{c_k} F_d + \frac{b_2 + b_m}{c_k} F_d + \frac{1}{c} F_d \right| \right)
$$

(47)

Then, $\dot{V}$ is negative outside the compact set. This demonstrates that the state $\mathbf{e}$ is uniformly bounded at the low force range.
4. Experimental results

In this section, the proposed control is applied to a real compliant actuator. The experimental setup is shown in Fig. 5. The system consists of the linear compliant actuator, two encoders, a motor driver and a PC with the controller. The compliant actuator is actuated by one brushless dc motor (MAXON 311537), capable of generating 1130 mNm stall torque. The first encoder (it is Encoder 1 in Fig. 5) is used to check the low force, while this encoder with other encoder (it is Encoder 2 in Fig. 5) is used to measure the high force. The motor driver is Elmo Harmonica 5/60 which can provide a maximum power output 200 W. The controller is the NI CompactRIO 9074 programmable automation controller that is an advanced embedded control and data acquisition system designed for applications that require high performance and reliability. In the controller, we use NI 9215 (Analog Input module), NI 9263 (Analog Output module), and NI 9516 (Encoder module) for the data acquisition. The entire control algorithm is written into two modules: monitoring (signal generator and program monitoring) and FPGA (control algorithm). The functional block-diagram of the entire control system is shown in Fig. 6. The sampling period for our test is chosen as 0.5 ms.

4.1. Model identification

For the control application, it is necessary to identify the model of the compliant actuator. From the analysis of Section 3, it is known that the model structure is a linear second order model which is described by a relationship from the input signal \( u \) to the output signal \( F_1 \). The system identification is to identify the parameters of the model from available input–output data. In the experiment, the model of the low force range is identified first. The input signal for the test consists of a square and 1 Hz which is around the working frequency. The input signal and the output force response of the open-loop actuator system are shown in Fig. 7. It should be noted that the input/output signals obtained are based on 32 bit FPGA and have no units. To illustrate their corresponding actual meanings, the second labels of the Y-axis are given in the figures. Based on the input/output signals, we use MATLAB to identify the model, where the least square identification algorithm is used. With the help of the System Identification Toolbox of MATLAB, the linear model of the actuator is given by

\[
\ddot{F}_1 = -208.3304F_1 - 11.838\dot{F}_1 + 197.22u
\]

In order to evaluate the identification performance, the variance-accounted-for factor is chosen as Bestfit model, which is defined as

\[
\text{Bestfit} = 100 \times \left( 1 - \frac{\text{var}(F_1 - \hat{F}_1)}{\text{var}(F_1)} \right)
\]

where \( \hat{F}_1 \) is the model output. The comparison of the simulated output and the actual measured output is shown in Fig. 8 and Bestfit in MATLAB Toolbox is 88.18. As observed, the model output is closed
to the measured actual output force, although they are slightly different at some points. The nonlinear model is given by

$$\hat{F} = A_1 F - B_1 u + B_1 [A_1 f(\dot{x}_1) + d_1]$$

with

$$F = \begin{bmatrix} F_1 \\ F_1 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 1 \\ -208.3304 & -11.8381 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 197.22 \end{bmatrix} \quad (49)$$

where $A_1 f(\dot{x}_1)$ is friction force and $d_1$ is the disturbance.

Next, the high force model is identified. Since the high force is large, square wave signal may damage the hardware due to frequent jumping. To prevent this occurrence, the identification for the high force range is based on the step response. The input and output signals used in the experiment are shown in Fig. 9. As indicated in the identification of the low force range, since the signals obtained have no units, the second label is also given in Fig. 9 to show the corresponding physical meanings. With the help of the System Identification Toolbox of MATLAB, the following linear model is obtained.

$$\hat{F}_1 = -1.3587 \times 10^6 F_1 - 6.6911 \times 10^3 F_1 + 1.1198 \times 10^6 u \quad (50)$$

The comparison of the simulated output and the actual measured output is shown in Fig. 10 and Bestfits in MATLAB Toolbox are 94.63. The model is given by

$$\hat{F} = A_2 F - B_2 u \quad (51)$$

with

$$A_2 = \begin{bmatrix} 0 & 1 \\ -1.3587 \times 10^6 & -6.6911 \times 10^3 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 \\ 1.1198 \times 10^6 \end{bmatrix} \quad (52)$$

4.2. Controller design

Based on the models obtained above, we design the low force and high force controllers according to the schemes proposed in Section 3.

Low force controller. According to our design, the low force is generated by the linear spring and its range is from 0 to 240 N. The PD controller is designed based on the LQR approach. The selection of the weighting matrix is $Q_1 = \text{diag}(700, 0.5)$ and $R_1 = 1$. Therefore, the PD gain becomes $\rho_1^{-1} B_1^T P_1$ which is obtained from (19), giving $K_p = 25.42, K_d = 0.81$. The eigenvalues of the linear closed-loop system are $(-5.92 \pm 13.16i)$ which are all negative. This implies that the designed PD feedback control system is stable in the sense of the linear model. On the other hand, we choose the following mode to compensate the friction and disturbance.

$$u_i = \mu_1 \text{sgn}(\dot{x}_1) + c_1 \text{sgn}(e_1 P_1 B_1) \quad (53)$$

The coefficient of the friction can be estimated by checking whether the force commences for a given a small amount of input signal; if it does not have, i.e., it is still zero, we have to increase the input signal until the force occurs. Fig. 11 shows the output force commences when the input signal is 850 which is the value of $\mu_1$. For the coefficient $c_1$ of the sliding mode, it can be estimated by checking whether the tracking performance is improved when a small value of $c_1$ is given; if no, increase its value until the performance is improved. Finally, it is determined by experimental test, that is 300. Then the proposed control law (23) is given by

$$u = 25.42 e_1 + 0.81 e_1 + 850 \text{sgn}(\dot{x}_1) + 300 \text{sgn}(e_1 P_1 B_1) + u_{g1} \quad (54)$$
where $u_{ff} = 1.056F_d + 0.06F_d + 0.0051F_d$.

High force controller. Taking a similar design as in the low force control, the PD control of the high force is also obtained based on the LQR approach. For given $Q_2 = \text{diag}(500, 0.000001)$ and $R_2 = 1$, the PD gains are $K_p = 21.18$, $K_d = 0.002$. The proposed control law is given by

$$u = 21.18e + 0.02e + u_{ff}$$

(55)

where $u_{ff} = 1.21333F_d + 0.006F_d + 8.93 \times 10^{-6}F_d$.

4.3. Performance assessment

In order to verify the effectiveness of the proposed control law, the actuator system is controlled to follow the force reference trajectory which is a sinusoidal signal. We will evaluate the control performance under the proposed low force control, high force control and switching control modes.

**Fig. 11.** Friction estimation.

**Fig. 12.** Tracking control performance of low force range 1 Hz.

**Fig. 13.** Tracking control performance of low force with a frequency of 2 Hz (the PD control).

**Fig. 14.** Tracking control performance of low force with a frequency of 2 Hz (the proposed control).
4.3.1. Low force tracking control

Using the proposed control law (54), Fig. 12 shows the control results, where the top figure is the output force response and the bottom one is the force tracking error. Thanks to the low-compliance spring, it is observed that the maximum tracking error achieved is very small, that is ±1.25 N. Alternative way to evaluate the control performance is based on the variance-accounted-for factor, checking the fidelity of the generated output force (readers can refer to [11] for the detailed content). The value for the force fidelity according to the given definition of [11] is 99.9%. As expected, the force profile generated by the low-compliance spring has very high accuracy.

To further evaluate the performance of the proposed controller, it is also compared with the PD control without the friction compensation and sliding mode scheme. The reference with a frequency of 2 Hz is used as a desired force profile. For a fair comparison, the PD control parameters are the same as the proposed controller. Fig. 13 shows the results using the PD control. With the same reference trajectory, the control performance using the proposed controller is also measured and the results are shown in Fig. 14. It is observed that the force control reaches to a maximum tracking error of 3 N by the PD control, compared to a maximum tracking error of 1.25 N when the proposed control scheme is used. This shows that the proposed controller can offer better control performance than that of the conventional PD control.

4.4. High force tracking control

The high force is generated by the torsional spring and it is greater than 240 N in our design. When entering a high force control, the proposed high force control (55) has to be used. Fig. 15 shows a high force control result with a frequency of 1 Hz. It can be seen that the force generated can track the desired force trajectory well. However, due to coarse force resolution which is 5.65 N, the output force has an error of about 15 N, about two times of the resolution. The force fidelity obtained for this tracking is 91.5%. Considering the high force range, the result is acceptable.

4.4.1. Switching control

In a practical situation, a more complicated case for the force profile is that it is involved in both low force and high force ranges. In this case, to track the force profile, we have to use the switching control, which is given by (34). Since the switching control is served for both force ranges. It is important to check whether or not the switching conditions (35), (36) hold. Here, it is found that the proposed low and high force controllers (54), (55) can achieve $\pi_1 > 0, \pi_2 > 0$. This implies that the designed switching control law can guarantee the stability of the closed-loop system. Fig. 16 shows the switching control results where SP is denoted as switching point.

**Fig. 15.** Tracking control performance of high force with a frequency of 1 Hz.

**Fig. 16.** Switching control results where SP is denoted as switching point.

**Fig. 17.** Ankle rehabilitation robot.
shows a switching force control result with a frequency of 1 Hz, where the switching point is $F_s = 240$ N. Initially, the low force control (54) is employed until the desired output force reaches to 240 N. Then, the switching control is activated and the force control enters the high force range. It is observed from the figure that a satisfactory result (<18 N) is achieved using the proposed switching law. This is also verified by measuring the force fidelity which is 97.7%. It can also be noted that at the low force range the signal is very smooth, but at high force range, the measured signal zigzags. This is because the force resolution at high force range is 5.65 N with the current encoder and torsional spring stiffness used in the design. From the figure, it can be seen that although the controller switches between low and high forces frequently, the closed-loop system is still stable. This further verifies our theoretical analysis in Theorem 3.

In summary, it can be concluded that: (1) the LQR-based PD parameter tuning is a good way to guarantee the closed-loop system performance and (2) switching control law is necessary to implement a stable transition between low and high force ranges.

4.5. Experiments with a human subject

Although the proposed actuator can generate accurate forces following a desired trajectory as shown in the experiments above, it is necessary to further test the rehabilitation robot with a human subject. The actuator presented in this paper was used to construct an ankle rehabilitation robot, as shown in Fig. 17.

In actual rehabilitation therapy, the desired force profile has to be determined based on the user needs at different gait phases. As the focus of this paper is on generating accurate forces from the actuator controlled, we use sinusoidal force signals as desired force trajectories for testing purpose.

The subject was asked to follow the force generated by the rehabilitation robot to do motions, as shown in Fig. 18. Since the load (the subject) is not fixed, i.e., $x_3 \neq 0$ (see Fig. 4), the generated output force is measured by using the linear potentiometer whose physical specifications are shown in Table 1. We tested the system performance at a frequency of 1Hz. Fig. 19 shows the control results, where the top figure is the output force response, the bottom figure is the force tracking error. It is observed that the desired force trajectory is tracked with a maximum tracking error of 6.5N which is increased when compared to the actuator control without human factor. This is because the human motions introduce some uncertainties and disturbances into the robot, which affect the control performance. But the control performance is acceptable as well as the closed-loop system is still stable.

5. Conclusion

We have presented a novel compliant actuator design and the control system for this actuator. By introducing one extra torsional
spring at the high force range, a truly compliant actuator with excellent force control fidelity. Theoretical analysis for the controller design has been given to guarantee the stability of the closed-loop system, especially for the switching control law. Experimental results have confirmed that the proposed controller can achieve good performance for the low, high force, and switching control. In the next stage of our research, we will collaborate with clinical partners to test our exoskeleton robot for gait rehabilitation.

References